

# Physical Determination of Relativistic Motion (Kinematics)

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**Abstract:** Special Relativity can be founded mathematically as an axiomatic system. This begins with abstract objects and postulates. However one can also consider what actually happens in measurements of real phenomena. The measurement-theoretically based view provides the mathematical formulation of abstract measurement results together with its physical conditions. In the interrelation of physical conditions (of classical laser ranging) the mathematical principle Lorentz symmetry is justified. In the contrast of physical conditions the limitations of the mathematical formalism become transparent regarding the physical resolution of the apparent Twin paradox. In the resulting formalism the physical meaning of its mathematical elements clarifies and simple principles of relativistic physics are uncovered - the key to overcome hidden stumbling blocks and apparent paradoxes from an (unscrutinized) classical intuition.

## 1 Foundation on principles of real measurements

Our objective is the foundation of Special Relativity. Usually this is done axiomatically. That is good, because in this way the whole mathematical formalism can be traced back to a manageable system of initial propositions which are logically independent from one another. Though this mathematical treatment of Physics already begins in the abstract. This stimulates questions for the physical meaning and the relation of this abstract mathematics to reality. For the foundation of basic physical notions one can also look into what actually happens in measurement practice. For example when asked for the meaning of time Theodor Hänsch - Nobel prize 2005 for inventing the optical frequency comb generator which facilitates the construction of most precise clocks - defines: 'Time is what one measures with a clock'. In his case a light clock.

He refers to the practical utilization of a manufactured clock, a finished product of watchmaking. We take up that thought and look at the process of watchmaking itself. We also

consider *methodical aspects* of the practice of physics. In this perspective one can understand by means of which actions an intrinsic observer arrives at watchmaking if he did not have it before. To provide this measurement instrument is an inevitable prerequisite for any quantification of durations in the first place.

Our measurement principle is based on classical measurements with light clocks. The global connection between distant measurement units is physically specified by the propagation of light. In classical laser ranging an observer can physically determine the motion of a measurement object. By setting up his measurement units and connecting them with light he produces a material model which (sufficiently precisely) reproduces the relative distance or duration to his object. In the actual measurement the real motion of his measurement object can be approximately replaced by his constructed model with light clocks. By means of this constructible substitution its real motion is physically specified. For the construction of those material models intrinsic observers always perform the same procedure of operations under the same (measurement) conditions. We introduce measurement termini which designate aspects of those physical substitutions precisely.

Step by step the notion of physical measures is constructed from their objective grounds. We reconstruct the formation of the theory. In the resulting formalism mathematical terms inherit physical meaning and their interrelation genetically unfolds. The measurement theoretical foundation provides the *mathematical formulation* of abstract measurement results together with its *physical conditions* {2}. From the *interrelation* of conditions of measurement actions the abstract postulate Lorentz symmetry is physically justified {3}. In the *contrast* of physical conditions the limitations of the mathematical formalism clarifies. In this way the apparent Twin paradox is physically resolved {4}. The benefit of this combined *way of thinking* in Physics is demonstrated on this example of Special Relativity Theory.

## 2 Intrinsic measurement practice

For the formation of colloquial notions - *motion* (of the object and of the observer himself), *space* and *rigid body* - from elementary sensual everyday experience we refer to Poincare [2] and Mach [4]. The fundamental idea of Poincare with regard to geometry is that *geometric properties* refer to characterizations of relative *motion* between *neighboring objects*. In the following we outline the development of a practice for the physical determination of those colloquial notions.

We begin from nothing but our experience with *moving objects* and *moving light*.<sup>1</sup> For illustration consider a (hidden) railway track along which *Alice*, *Bob* and *Otto* only address their relative motion as well as the motion of light (figure 1a). Observed objects and observers are in motion. For practical purposes like navigation in these circumstances they specify their relative motion more precisely.

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<sup>1</sup>We regard 'light' as electromagnetic interaction between charged objects. Between two distant mirrors the propagation of electromagnetic effects spreads finite. Its homogenous and isotropic way of propagation is a metrical specification which is found provided classical metric reference devices are constructed.

On the basis of everyday experience *Alice* and *Bob* roughly know what is meant by 'length' and 'duration' {2.1}. The goal is to render these (colloquially phrased) notions more precisely by physical measurements. Intrinsic observers perform basic measurement operations without the need of an absolute reference space in the background or coordinate frames and without any mathematical presupposition.

Intrinsic measurements directly relate to actual circumstances *Alice* and *Bob* face from their respective perspective. We presuppose that every observer can produce 'straight' 'rigid' rulers and 'uniform running' clocks by himself. The *construction* of measurement instruments and the *convention* of rules for their proper use in measurement operations is the product of a historical development. From empirical knowledge on feasibilities and on outcomes of *actions* in everyday work we arrive (circularity free and without mathematical presuppositions) at the formation of classical measurement practice {2.2}. In this naturalist conception [15] we assume that on the basis of local intrinsic work experience every observer has arrived at the classical Euclidean metric [14].

In starting snapshot figure 1a we illustrate observed objects and observers in motion. After including the historical dimension of work experience *Alice* and *Bob* arrive - provided their intrinsic and local classical metric - at the kinematics of Special Relativity Theory: Each observer measures *Otto's* relative motion by means of classically constructed light clocks {2.4}, by following rules for the connection of these measurement units {2.5} and with reference to the motion of light (classical light principle<sup>2</sup>) {2.3}. All they can do (and for their intrinsic purposes that is all they need) is to specify - by means of operating with those devices - their relative motion relative to the observable motion of light.

## 2.1 Working observer

We approach measurement practice based on the experience from work and experiment. There we obtain our knowledge of realizable behavior of natural objects and there we also rehearse expedient procedures of handling them. *Alice* and *Bob* do not work with isolated objects; they begin working with real objects in the real world. The way their work object behaves depends on external conditions (some are known and others not even discovered). Real work objects - including the raw materials to produce measurement devices - are subject to external conditions. *Alice* and *Bob* acquire empirical knowledge about the interrelation of those conditions. Their work object realizes an *actual* property if all necessary conditions are provided (by external objects). Otherwise those interrelations remain *possible* properties. Their pre-scientific knowledge of everyday work experience is expressed in familiar everyday language. In measurement practice *Alice* and *Bob* physically specify that acquired knowledge of realizable behavior of natural objects. Their skills of producing measurement devices and utilizing them to examine natural phenomena scientifically is based on and successively grows with their local intrinsic work experience.

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<sup>2</sup>The propagation of light represents highly reliably a reference motion which is 'equal', 'straight' and independent from the source. This metrical characterization of light is likewise a discovery possible not before a developed classical metric.

From such basis we presuppose demonstrable (work) objects and their relations (worked out under given conditions). So far observers express their knowledge of that demonstrable behavior in colloquial language (e.g.: *Alice*, *Bob* and *Otto* are moving relative to one another. They exchange signals of light. etc.). The usage of common denominations ('*Alice*', '*Otto*', 'move', 'distance') with their common meaning is presupposed as a known part of work experience.<sup>3</sup>

Leibniz characterizes the notions of 'space' and 'time' as relations between the observable things. *Space* brings order into things which happen simultaneously. *Time* brings order into things which happen sequentially. We distinguish the length of an object (resp. the distance between its end marks) and the duration of ongoing processes. Our goal is to render that order of co-existence and succession more precisely. We want to physically specify the 'form' of relative motion and the 'extent' of distances and durations.

In a measurement one determines real measurement objects by means of equally real measurement instruments. Pre-theoretically these natural objects are presupposed from work experience as known representatives of (conditionally) realizable properties. Before their physical determination both the potential measurement objects and the raw materials for (still to be produced) measurement instruments are identified by demonstration. We describe our knowledge from working with them purely colloquially.

## 2.2 Classical metric

For the constructive explanation of measurements in physics we refer to the action-theoretical conception *Protophysics* of the Erlangen School by Lorenzen and Janich [14]. It provides an understanding of the historical development of fundamentals of physics. How does one arrive at the concept of measured quantities, if one does not have them? The quantification of colloquially described relations is explained circularity free in the categories of purpose and expedient means in everyday work. Before observers *Alice* or *Bob* can specify the motion of *Otto* by means of operating with measurement devices they have to build them. How do they get there before they were known?

In the production of measurement instruments one picks up raw materials from the natural environment, works on them and reshapes them for practical needs. The success of (tentative) manufacturing methods for rulers and clocks - as admissible devices to specify

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<sup>3</sup>The meaning of colloquial expressions (of work experience) stems from the activity of exemplary demonstration (of sufficiently constant phenomena). We can neither demonstrate pure matter in isolation from its behavior nor pure behavior detached from matter. Matter and its (conditionally realized) behavior are inseparably unified. The smallest unit of meaning - reflecting this unity - in colloquial speech are simple sentences. Colloquially we express demonstrable facts in simple sentences like 'Otto is long'. Therein the subject Otto  $\mathcal{O}$  and the attribute length  $l$  are distinguishable but inseparably unified [9] [13]. The subject terminus 'long Otto' emphasizes the subject  $\mathcal{O}$  which embodies the property long  $l$  - in short the *long Object* symbolized by  $\mathcal{O}_l$ . The predicate terminus 'Otto's length' emphasizes the property which Otto represents, in short the *objective Length* symbolized by  $l_{\mathcal{O}}$ . These basic termini enter the colloquial description of measurement practice. An observer operates (intrinsically) with measurement objects  $\mathcal{O}_l$  to physically specify their objective measures  $l_{\mathcal{O}}$ .

pre-theoretically purely colloquially described properties 'length' and 'duration' - is secured by *test procedures*. They involve guidelines for the examination of the *straight form* of a constructed ruler and of the *uniform running* of a clock.<sup>4</sup> In the protophysical foundation of Euclidean Geometry [14] the testing method for geometrical shapes originates from the practice of grinding processes: To test whether a manufactured body has a 'flat' surface one has to produce two moldings of that body and check if one can fittingly (!) shift their two imprint surfaces against one another. If between the two moldings is a gap continue grinding them against one another; make two new moldings and check again. Similarly if one has manufactured a body with two flat surfaces which intersect one another, then the intersecting edge represents a 'straight' line. Such test norms originate from intuitively controlled actions in everyday (technical) work. These test procedures are rehearsed in work practice which is governed by the rationality of purpose and expedient means. They are likewise specifiable purely in colloquial language. These expedient *work-norms* are being *explicated as measurement norms* [15] for the purpose of physically specifying known work conditions [12]. In this way one can understand how practicable rules of pre-scientific technical behavior develop into norms for measurement operations and their theoretical description.

In the art of watchmaking for example the watchmaker evaluates by test procedures for uniform running whether his tentatively produced clock realizes sufficiently precise a uniform motion. In the empirical interplay - of analyzing working conditions and examining respectively achieved work products - the *manufacturing method* is continually refined until the produced clock *realizes* in sufficiently good approximation the aspired *ideal* of uniform motion. In this process we make the practical experience that the ideal is never completely realizable. The closer one wants to approach the more effort and workload is required in the production and also for the conservation of the produced device.

*Measurement instruments* are produced as sufficiently constant (for practical purposes) representatives of - until now colloquially phrased notions - 'length' and 'duration'. For this reason clocks and rulers are to be understood - not simply as arbitrary designations of natural objects but - as *artifacts*: Measurement instruments are produced as a product of norm realizing manufacturing actions [15].

In the practice of measuring the 'distance' - e.g. to *Otto's* relative position - the attribute 'physical' refers to operations which *Alice* or *Bob* realize with their measurement instruments. These artifacts are connected in measurement operations by the observer. They *connect* their manufactured rulers  $\mathbf{1}_s$  side by side in a straight way until the constructed assembly - symbolized by  $\mathbf{1}_s * \dots * \mathbf{1}_s$  - reproduces (with sufficient precision) the real 'length'  $s$  of measurement object  $\mathcal{O}$

$$\mathbf{1}_s * \dots * \mathbf{1}_s =_s \mathcal{O} \quad .$$

*Alice* or *Bob* *physically specify* the real length of measurement object  $\mathcal{O}$  by producing a *ma-*

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<sup>4</sup>Before *Bob* can specify the form of *Otto's* relative motion he has to find out if his own clock provides a uniform ticking reference. Before *Alice* can determine whether *Otto's* nose is crooked she needs to know if her ruler provides a straight reference by itself. Those test rules for the admissability of measurement devices do *not* presuppose an already existent *prototype* for straight lines and an ur-clock which one can simply copy or transport. With those test rules in mind measurement devices are produced in the first place.

*terial model*. In the model they can count the number of the connected measurement units  $\mathbf{1}_s$ . Only by means of this *constructible substitution* those formerly colloquially phrased notions 'length' and 'duration' are metricized.

A device - manufactured according to the test procedures of uniform motion - provides external conditions which allow for a uniform straight *form* of motion. By metricizing the length of the traversed stretch of way (of the moving clock hand) one obtains a metrical measurement instrument for 'durations'.

A watchmaker successively reduces disturbing effects from *external* sources according to *protophysical* - manufacturing orienting - *norms* of watchmaking. Sufficient shielding is achieved when the manufactured product satisfies the test procedure: Take two structurally identical copies of the clock and align them such that their clock hands are running straight into fixed (e.g. perpendicular) directions. Then one can couple the motion of the two clock hands e.g. by a mechanical transmission; draw down the stretch of way of their superposed motion and *check* its geometric form. The clocks realize *uniform running* if - independently from when each of the two clocks was started and coupled together - their superposed stretch of way always has the form of a straight line. Again the problematic path realizes the ideal of a straight line if any two segments of it can be fittingly (!) shifted against one another.<sup>5</sup> In this case the manufactured clock (sufficiently) realizes the ideal of uniform running. The *ideal* of such a motion - which is free from disturbing effects of external influences (according to these manufacturing guidelines) - we call isolated motion.

In practice one cannot manufacture clocks which run (approximately) uniform for unlimited durations due to dissipative effects of accumulated friction etc. Instead one can produce devices which satisfy the test norm at least within a limited framework. A device - where under repeated conditions the clock handle runs through a fixed stretch of path with finite length - represents a measurement unit  $\mathbf{1}_t$  for 'durations'.

In a measurement operation for the 'duration' of process  $\mathcal{P}$  these artifacts are connected by the observer. They connect the uniform ticks of their manufactured clocks  $\mathbf{1}_t$  directly one after another until the composed process - symbolized by  $\mathbf{1}_t * \dots * \mathbf{1}_t$  - reproduces (with sufficient precision) the 'duration'  $t$  of measurement process  $\mathcal{P}$

$$\mathbf{1}_t * \dots * \mathbf{1}_t =_t \mathcal{P} \quad .$$

An observer *physically specifies* the real duration of measurement process  $\mathcal{P}$  by producing a *material model*. In his model he can count the number of connected measurement units  $\mathbf{1}_t$ . Only by this *constructible substitution* our common notion of 'duration' is metricized. In this way we arrive at the classical Euclidean metric for space and time.

We grasp 'measurements' as a physical specification of actual properties. Each observer determines the (conditionally realized) property of the measurement object by means of

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<sup>5</sup>By this procedure the protophysical norm to test for *uniform* forms of motion is based upon the test for the *straight* geometric form of rigid objects.

**constructible substitution.** The physical specification begins with the colloquial expression of work experience. We use everyday language and words with their everyday meaning. For technical termini of measurement practice we give a precise definition. In the following physical determination of relativistic motion we explain aspects of this actual substitution, introduce operational denominations and their mathematical formulation. The *formalism* of Special Relativity Theory is substantiated on the basis of colloquially expressed knowledge of everyday work experience. In this way one can understand the relation of this abstract mathematics to reality. The validity of mathematical terms and of their genetic relations is justified by our life-world based definition of measurement termini.

## 2.3 Light principle

Under the conditions of classical measurements the motion of light can be specified metrically. The propagation of light from various sources is physically determined by means of classical measurement devices. In general these objects move relative to the light source as well. Provided classical metric we know that: (i) no object is faster than free light, (ii) the motion of light is independent from the source and its state of motion and (iii) once two nearby sources *Alice* and *Bob* have emitted free moving light towards *Otto*, locally neither of the two rays  $\overline{AO}$  and  $\overline{BO}$  overtakes or deviates from the other. Thus free moving light provides a universal reference for any intrinsic observer.

The associated processes and their interrelation are depicted in a *spacetime diagram* (see figure 1b). It portrays the unified (objective) motion for all relevant (moving) objects.<sup>6</sup> *Alice*, *Bob* or *Otto* can move equally or not, but they cannot overtake their light. The light *Alice* or *Bob* once sent to *Otto* propagates independently no matter how they move. If *Alice* sends her light to *Otto* and shortly after it passes *Bob* he sends his own light to *Otto* as well, then both light rays  $\overline{AO}$  and  $\overline{BO}$  neither overtake or deviate from one another. Classical rulers and clocks approximate a straight line and uniform motion. Free light propagates locally - i.e. verifiable by measurements with these manufactured (local) classical devices - in a uniform and straight way. In a spacetime diagram we represent this form of motion by a *straight line*. Locally the light *Alice* or *Bob* send to *Otto*  $\overline{AO}$  and  $\overline{BO}$  remains parallel or congruent.

By means of classical metric - i.e. within the domain of classical measurements of length and duration - we discover: *Locally* free light represents a *uniform, isotropic and straight form of motion*. This '*Light principle*' is taken as a basis for laser ranging measurements. This hypothesis we presuppose also along *global paths of light* which can be thought of as a connected covering of many local segments.

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<sup>6</sup>This depiction faithfully portrays the interrelations between various motions according to the discovered Light principle. But such *pictorial description* - of basic empirical facts - is not a *physical determination* by means of precise measurements. Their description in terms of physical measures is not yet obtained.

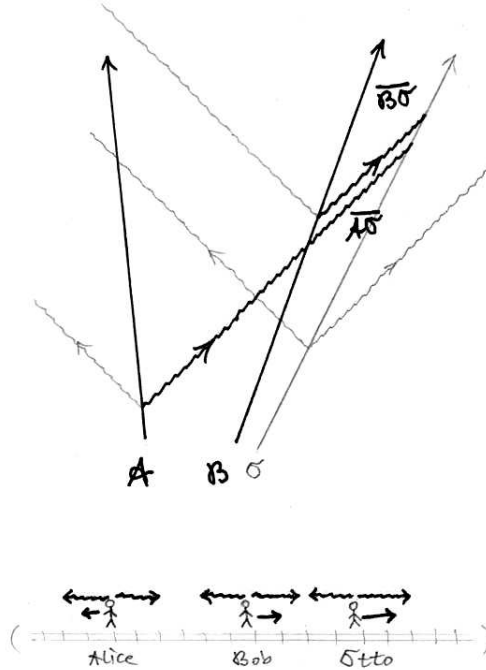


Figure 1: a) moving objects and moving light b) interrelation of corresponding processes

## 2.4 Light clock

In order to give physical meaning to the concept of time - Einstein demands - requires the use of some process which establishes relations between distant locations. In principle one could use any type of process. Most favorable for the theory one chooses a process about which we know something certain. For the free propagation of light this holds much more than for any other process [5].

Because of the universal Light principle 'laser ranging' is a reliable *practice of navigation*.<sup>7</sup>

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<sup>7</sup>This method is not an invention of Special Relativity or Classical Mechanics. Before Einstein or Newton this practice was developed in real life. Throughout millennia of evolution bats coordinating their living at night or dolphins hunting under invisible conditions discovered and rehearsed the possibility of (i) *producing* sonic waves and (ii) exploiting that *tool* to master survival under their living conditions.

Only provided the development of the classical metric we understand why it works so reliably in practice. For the duration of sonar ranging acts the emitting organism sufficiently represents a rigid body with constant state of motion. Sound propagates sufficiently straight and uniform. Furthermore the light principle is not a necessary condition for successful maneuvering of bats and dolphins within an environment of comparably small relative motions (compared with the much larger speed of sound waves in air and water).

The specification of actions in our (work) practice is constitutive for the formation of basic physical concepts. We can *physically specify* the *conditions* of that already existent technique of navigation. Upon Einstein's discovery of the Light principle we begun scrutinizing the conditions of our measurement practice; providing the basis for the formation of his (practically meaningful) physical theory. By means of this physical specification of common navigation techniques we step beyond simple bats and dolphins. Its theoretical



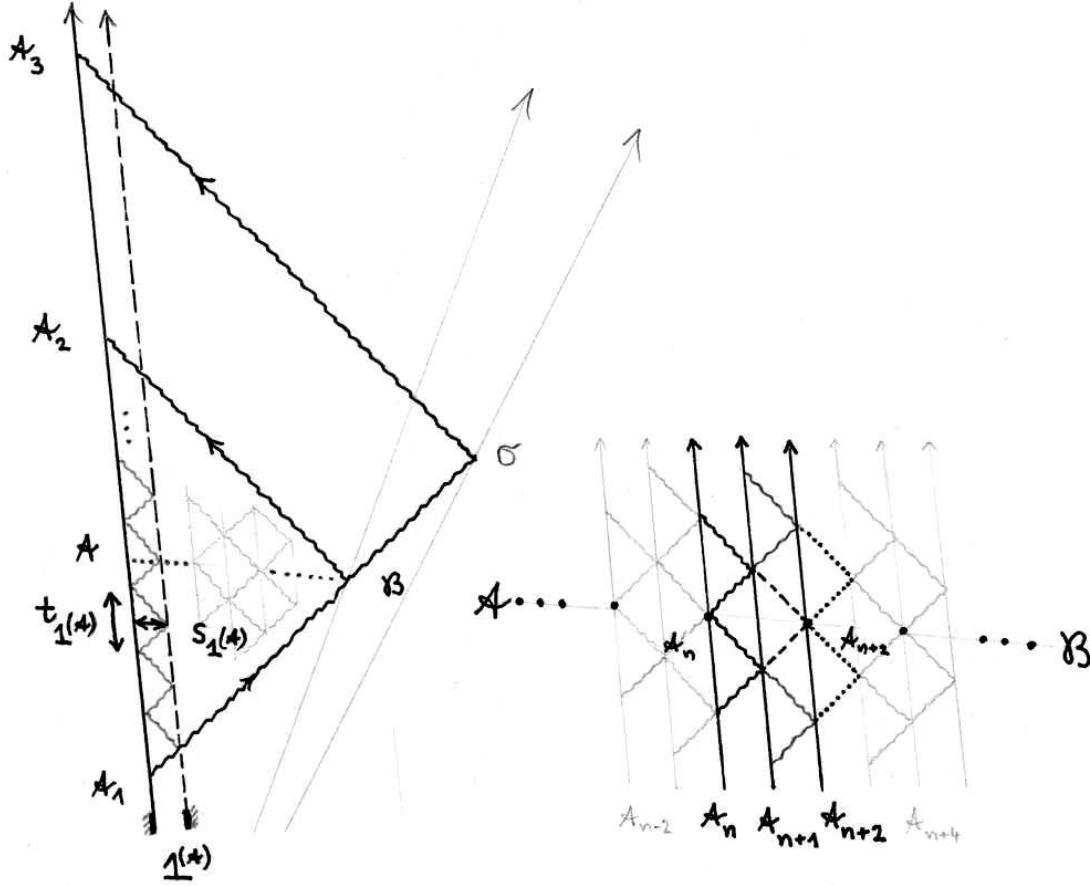


Figure 2: a) laser ranging b) consecutive and adjacent connection of light clocks

When for Alice the objects of interest Bob and Otto are out of reach she sends out light towards Bob  $A_1 \rightsquigarrow B \rightsquigarrow A_2$  and towards Otto  $A_1 \rightsquigarrow O \rightsquigarrow A_3$  and waits until their reflection returns (see figure 2a). In radar round trips we focus on the distance traveled and Alice waiting time. For a pair of light cycles  $A_1 \rightsquigarrow B \rightsquigarrow A_2$  and  $A_1 \rightsquigarrow O \rightsquigarrow A_3$  Alice notices the order in which the light returns. She notes if waiting times  $t_{\overline{A_1 A_2}}$  and  $t_{\overline{A_1 A_3}}$  are equal or different from one another. By the Light principle a longer waiting time for the independently moving light corresponds to a larger distance traveled  $s_{\overline{AB}}$  resp.  $s_{\overline{AO}}$  from Alice to the turning point Bob resp. Otto and back.

By means of laser ranging with closed light cycles  $A_1 \rightsquigarrow B \rightsquigarrow A_2$  Alice obtains an ordering relation with respect to her round trip waiting time. Provided classical metric Alice can determine the extent of *local* distances and durations quantitatively. By introducing a light clock Alice provides an intrinsic reference device to measure *global* distances  $s_{A_1 \rightsquigarrow B \rightsquigarrow A_2}$  and

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conception enabled engineers to trigger a revolution of technical applications (GPS-satellites, synchronization and coordination of global partition of work, police radar traps, Lunar-Laser-Ranging etc.).

durations  $t_{\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2}$  as well.

Alice constructs her *light clock*  $\mathbf{1}^{(\mathcal{A})} : \mathcal{L}_I \rightsquigarrow \mathcal{L}_{II} \rightsquigarrow \mathcal{L}_I \dots$  with two nearby mirrors  $\mathcal{L}_I$  and  $\mathcal{L}_{II}$  which are kept at constant distance through a rigid frame. She constructs the rigid frame according to protophysical manufacturing guidelines and on the basis of classical metric both mirrors can be aligned parallel. Between both mirrors the light constantly oscillates back and forth. Each tick of this new *measurement unit*  $\mathbf{1}^{(\mathcal{A})}$  covers the same fixed *unit distance*  $s_{\mathbf{1}^{(\mathcal{A})}}$  and because of (her classical metrical knowledge of) the Light principle each successive tick takes the same *unit time*  $t_{\mathbf{1}^{(\mathcal{A})}}$ . Alice can express the independent speed of the light  $c$

$$c \cdot t_{\mathbf{1}^{(\mathcal{A})}} := 2 \cdot s_{\mathbf{1}^{(\mathcal{A})}} \quad (1)$$

by means of her measurement unit  $\mathbf{1}^{(\mathcal{A})}$  in terms of unit time  $t_{\mathbf{1}^{(\mathcal{A})}}$  and unit distance  $s_{\mathbf{1}^{(\mathcal{A})}}$ .

Alice (classically constructed) light clock replaces her formerly manufactured rulers and clocks. The protophysical test norms - for manufacturing both traditional clocks and light clocks - remain the same. The older clocks (i.e. the product of former clock making) are only replaced by new light-clocks because the latter realize the aspired ideal of uniform running more precisely. This new measurement instrument together with the (classically determined) Light principle provides a universal representative for the local classical metric and Euclidian geometry. On this basis the motion of light is not anymore measured by means of classically produced rulers and clocks; instead all other motions are determined with respect to the motion of light and (classical protophysically constructed) light clocks. This *paradigm shift* concerns the former priority of classical measurement devices over the propagation of light. The motion of light becomes a measurement standard itself.

With this practice we can realize a new measurement principle: The classical (metrically constructed) light clock is a new *measurement unit*. They are independent representatives for the local classical metric. For the physical specification of a measurement object - e.g. of Bob's relative motion - we *successively connect* many neighboring light clocks by means of the independent propagation of light between them. By means of the connecting light the isolated local notions of classical metric (each represented by a respective light clock) are successively joined into their global interrelation.

## 2.5 Direct connections

### 2.5.1 Time-like

Now Alice can join together light clock ticks  $\mathbf{1}^{(\mathcal{A})}$  *one after another* until the sequence  $\mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})}$  reproduces the duration of her laser ranging interval  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$

$$\overline{\mathcal{A}_1 \mathcal{A}_2} =_t \mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})} \quad . \quad (2)$$

Round trip interval and sequence of ticks have equal duration  $t_{\overline{\mathcal{A}_1 \mathcal{A}_2}} = t_{\mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})}}$ . In the constructed sequence  $\mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})}$  Alice counts the number of ticks  $\mathbf{1}^{(\mathcal{A})}$  (representing unit time  $t_{\mathbf{1}^{(\mathcal{A})}}$ ) which we symbolize  $\sharp \{ \mathbf{1}^{(\mathcal{A})} \} =: \left\{ \frac{t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}}{t_{\mathbf{1}^{(\mathcal{A})}}} \right\} \equiv t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}^{(\mathcal{A})}$ . Alice physically specifies

the duration of her laser ranging interval  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}}$  by means of counting units in her produced sequence of light clock ticks. Alice resulting *measurement product* we call *physical measure*

$$t_{\overline{\mathcal{A}_1\mathcal{A}_2}} \stackrel{(2)}{=} t_{\mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})}} =: \left\{ \frac{t_{\overline{\mathcal{A}_1\mathcal{A}_2}}}{t_{\mathbf{1}^{(\mathcal{A})}}} \right\} \cdot t_{\mathbf{1}^{(\mathcal{A})}} \equiv t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(\mathcal{A})} \cdot t_{\mathbf{1}^{(\mathcal{A})}} \quad . \quad (3)$$

To measure **duration**  $t$  of laser ranging interval  $\overline{\mathcal{A}_1\mathcal{A}_2}$  (her measurement *object*) Alice utilizes a light clock  $\mathbf{1}^{(\mathcal{A})}$  (her measurement *unit*) to consecutively construct a sequence of light clock ticks  $\mathbf{1}_t^{(\mathcal{A})} * \dots * \mathbf{1}_t^{(\mathcal{A})}$  (we call the material model measurement *means*). Measurement practice affirms that Alice laser ranging waiting time  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}}$  (the objective measure) and the duration of her constructed sequence of ticks (mean measure) are equal  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}} = t_{\mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})}} =: t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(\mathcal{A})} \cdot t_{\mathbf{1}^{(\mathcal{A})}}$  (this is her intrinsically constructed *physical measure*).

### 2.5.2 Space-like

Furthermore Alice can join together ticking light clocks  $\mathbf{1}^{(\mathcal{A})}$  *side by side* until the layout  $\mathbf{1}_s^{(\mathcal{A})} * \dots * \mathbf{1}_s^{(\mathcal{A})}$  reproduces the length of her laser ranging route  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$

$$\overline{\mathcal{AB}} =_s \mathbf{1}^{(\mathcal{A}_1)} * \dots * \mathbf{1}^{(\mathcal{A}_n)} \quad . \quad (4)$$

Laser ranging path and layout of light clocks have equal length  $s_{\overline{\mathcal{AB}}} = s_{\mathbf{1}^{(\mathcal{A}_1)} * \dots * \mathbf{1}^{(\mathcal{A}_n)}}$ . To lay out the radar path  $\overline{\mathcal{AB}}$  imagine a moving swarm of identical light clocks  $\mathbf{1}^{(\mathcal{A}_i)} : \mathcal{L}_i \rightsquigarrow \mathcal{L}_{i+1} \rightsquigarrow \mathcal{L}_i \dots$  with  $i = 1 \dots n$ . Beginning with her own light clock  $\mathbf{1}^{(\mathcal{A})} \equiv \mathbf{1}^{(\mathcal{A}_1)}$  Alice successively places pairs of light clocks  $\mathbf{1}_s^{(\mathcal{A}_i)}$  and  $\mathbf{1}_s^{(\mathcal{A}_{i+1})}$  next to one another in an intrinsically *simultaneous and straight way* (see figure 2b):

- (a) Suppose we have successively laid out light clocks from  $\mathbf{1}^{(\mathcal{A}_1)}$  all the way to  $\mathbf{1}^{(\mathcal{A}_n)}$ . Consider the two ticks of light clock  $\mathbf{1}^{(\mathcal{A}_n)} * |_{\mathcal{A}_n} \mathbf{1}^{(\mathcal{A}_n)}$  around the moment  $\mathcal{A}_n$ .
- (b) The next moving light clock  $\mathbf{1}^{(\mathcal{A}_{n+1})}$  has to be placed such that the (dashed) extension of light from  $\mathbf{1}^{(\mathcal{A}_n)} * |_{\mathcal{A}_n} \mathbf{1}^{(\mathcal{A}_n)}$  is physically identical with  $\mathbf{1}^{(\mathcal{A}_{n+1})}$ .
- (c) Then - by the isotropy of Light - starting from  $\mathcal{A}_{n+1}$  the light travels in the identical *round trip duration*  $t_{\mathbf{1}^{(\mathcal{A}_{n+1})}}$  the same distance to the left (back to  $\mathcal{A}_n$  inside previous light clock  $\mathbf{1}^{(\mathcal{A}_n)}$ ) as to the right (forward to  $\mathcal{A}_{n+2}$  inside new light clock  $\mathbf{1}^{(\mathcal{A}_{n+1})}$ ).<sup>8</sup>
- (d) If that is satisfied by light clock  $\mathbf{1}^{(\mathcal{A}_{n+1})}$  consider the sequence which includes the preceding and the following tick  $\mathbf{1}^{(\mathcal{A}_{n+1})} * \mathbf{1}^{(\mathcal{A}_{n+1})} * \mathbf{1}^{(\mathcal{A}_{n+1})}$ .
- (e) The next moving light clock  $\mathbf{1}^{(\mathcal{A}_{n+2})}$  has to be placed such that the (dotted) extension of light from  $\mathbf{1}^{(\mathcal{A}_{n+1})} * \mathbf{1}^{(\mathcal{A}_{n+1})} * \mathbf{1}^{(\mathcal{A}_{n+1})}$  is physically identical with the two ticks of light clock  $\mathbf{1}^{(\mathcal{A}_{n+2})} * |_{\mathcal{A}_{n+2}} \mathbf{1}^{(\mathcal{A}_{n+2})}$  around the moment  $\mathcal{A}_{n+2}$ .

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<sup>8</sup>When light clock  $\mathbf{1}^{(\mathcal{A}_{n+1})}$  measures adjacent light clock  $\mathbf{1}^{(\mathcal{A}_n)}$  to its left then  $s_{\mathbf{1}^{(\mathcal{A}_n)}}^{(\mathcal{A}_{n+1})} \cdot s_{\mathbf{1}^{(\mathcal{A}_{n+1})}} \equiv -s_{\mathbf{1}^{(\mathcal{A}_{n+1})}}$ .

- (f) By analogous induction steps  $\mathbf{1}^{(\mathcal{A}_n)} * |_{\mathcal{A}_n} \mathbf{1}^{(\mathcal{A}_n)} \Rightarrow \mathbf{1}^{(\mathcal{A}_{n+1})} \Rightarrow \mathbf{1}^{(\mathcal{A}_{n+2})} * |_{\mathcal{A}_{n+2}} \mathbf{1}^{(\mathcal{A}_{n+2})} \forall n$  Alice successively proceeds all the way towards  $\mathcal{B}$ ob.

In every step the (dashed resp. dotted) identical extension of independently moving light connects adjacent light clocks. This physically realizes a straight comoving *connection* of light clocks  $\mathbf{1}^{(\mathcal{A})}|_{\mathcal{A}} * \mathbf{1}^{(\mathcal{A}_2)} * \mathbf{1}^{(\mathcal{A}_3)}|_{\mathcal{A}_3} \dots \mathbf{1}^{(\mathcal{A}_n)}|_{\mathcal{A}_n} * \mathbf{1}^{(\mathcal{A}_{n+1})} * \mathbf{1}^{(\mathcal{A}_{n+2})}|_{\mathcal{A}_{n+2}}$ . Along the connecting moments  $\mathcal{A}, \mathcal{A}_3 \dots \mathcal{A}_n, \mathcal{A}_{n+2} \dots \mathcal{B}$  these light clocks tick synchronized.

This is how Alice intrinsically constructs by means of light clocks a straight spacelike path. In laser ranging that straight path connects Alice to her distant measurement object Bob. This constructed adjacent layout of comoving light clocks  $\mathbf{1}_s^{(\mathcal{A}_1)} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)}$  represents Alice *simultaneous straight measurement path* towards Bob  $\overline{\mathcal{AB}}$ .

Alice measures the length along her straight connecting path  $\overline{\mathcal{AB}}$  by means of her constructed layout of light clocks  $\mathbf{1}_s^{(\mathcal{A}_1)} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)}$ . She counts the number of ticking light clocks  $\mathbf{1}_s^{(\mathcal{A}_i)}$  (each representing unit distance  $s_{\mathbf{1}^{(\mathcal{A})}}$ ) which we symbolize  $\sharp \left\{ \mathbf{1}_s^{(\mathcal{A}_i)} \right\} =: \left\{ \frac{s_{\overline{\mathcal{AB}}}}{s_{\mathbf{1}^{(\mathcal{A})}}} \right\} \equiv s_{\overline{\mathcal{AB}}}^{(\mathcal{A})}$ . Alice resulting measurement product is the direct *physical measure*

$$s_{\overline{\mathcal{AB}}}^{(\mathcal{A})} = s_{\mathbf{1}^{(\mathcal{A}_1)} * \dots * \mathbf{1}^{(\mathcal{A}_n)}} =: \left\{ \frac{s_{\overline{\mathcal{AB}}}}{s_{\mathbf{1}^{(\mathcal{A})}}} \right\} \cdot s_{\mathbf{1}^{(\mathcal{A})}} \equiv s_{\overline{\mathcal{AB}}}^{(\mathcal{A})} \cdot s_{\mathbf{1}^{(\mathcal{A})}} \quad . \quad (5)$$

To directly measure the **length**  $s$  along her laser ranging path  $\overline{\mathcal{AB}}$  (the measurement *object*) Alice utilizes light clocks  $\mathbf{1}^{(\mathcal{A})}$  (i.e. the same measurement *units*) - though now to adjacently produce a layout of ticking light clocks  $\mathbf{1}_s^{(\mathcal{A}_1)} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)}$  (her new measurement *means*). Measurement practice affirms that Alice laser ranging distance  $s_{\overline{\mathcal{AB}}}$  and the length of her produced layout of light clocks are equal  $s_{\overline{\mathcal{AB}}} = s_{\mathbf{1}^{(\mathcal{A}_1)} * \dots * \mathbf{1}^{(\mathcal{A}_n)}} =: s_{\overline{\mathcal{AB}}}^{(\mathcal{A})} \cdot s_{\mathbf{1}^{(\mathcal{A})}}$  (her intrinsically constructed *physical measure*).

### 2.5.3 Spacetime-like

In elementary laser ranging  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  Alice determines the relative position of Bob by both space- and time-like connection. From moment  $\mathcal{A}_1$  Alice physically specifies his relative location at moment  $\mathcal{B}$  when the radar pulse reflects (see figure IIa) by means of

1. the consecutive interval of (light clock) ticks until the 'half-time' moment  $\mathcal{A}$

$$\overline{\mathcal{A}_1 \mathcal{A}} := \mathbf{1}_t^{(\mathcal{A})}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(\mathcal{A})} \stackrel{\text{step (f)}}{=} \frac{1}{2} \cdot \left\{ \frac{t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}}{t_{\mathbf{1}^{(\mathcal{A})}}} \right\} \cdot \mathbf{1}_t^{(\mathcal{A})}$$

2. and directly connected in the (light clock) tick  $\mathbf{1}_t^{(\mathcal{A})}|_{\mathcal{A}} \equiv \mathbf{1}_s^{(\mathcal{A}_1)}|_{\mathcal{A}}$  at moment  $\mathcal{A}$  by means of the adjacent layout of (ticking) light clocks until the moment  $\mathcal{B}$

$$\overline{\mathcal{AB}} := \mathbf{1}_s^{(\mathcal{A}_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)} \quad .$$

The resulting layout - of *Alice consecutive and adjacent connection* of light clocks  $\mathbf{1}^{(A)}$  - reproduces her spatiotemporal distance towards *Bob*

$$\overline{\mathcal{A}_1\mathcal{B}} \stackrel{(6)}{=}_{t,s} \overline{\mathcal{A}_1\mathcal{A}} * \overline{\mathcal{A}\mathcal{B}} \stackrel{(6)}{=}_{t,s} \mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \underbrace{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}}}_{\equiv \mathbf{1}_s^{(A_1)}|_{\mathcal{A}}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}} \quad . \quad (6)$$

Now *Alice* regards the *individual aspects* duration (3) and length (5) of her elementary laser ranging practice *unified*. To directly measure the spatiotemporal distance  $(t, s)$  of her laser ranging path  $\overline{\mathcal{A}_1\mathcal{B}}$  (the measurement *object*) *Alice* utilizes light clocks  $\mathbf{1}^{(A)}$  (as a *unified* spatiotemporal measurement *unit*) to consecutively and adjacently construct a layout of ticking light clocks  $\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(A_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}$  (her spatiotemporal measurement *means*) until the spatiotemporal distance of her laser ranging path  $\overline{\mathcal{A}_1\mathcal{B}}$  is reproduced. Along the consecutive segment  $\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(A_1)}|_{\mathcal{A}}$  she counts the number of light clock ticks  $\mathbf{1}_t^{(A)}$  (each representing unit time  $t_{\mathbf{1}^{(A)}}$ ) which we symbolize  $\sharp\{\mathbf{1}_t^{(A)}\} =: \left\{ \frac{t_{\overline{\mathcal{A}_1\mathcal{B}}}}{t_{\mathbf{1}^{(A)}}} \right\} \equiv t_{\overline{\mathcal{A}_1\mathcal{B}}}^{(A)}$  and along adjacent segment  $\mathbf{1}_s^{(A_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}$  she counts the number of ticking light clocks  $\mathbf{1}_s^{(A)}$  (each representing unit distance  $s_{\mathbf{1}^{(A)}}$ ) which we symbolize  $\sharp\{\mathbf{1}_s^{(A)}\} =: \left\{ \frac{s_{\overline{\mathcal{A}_1\mathcal{B}}}}{s_{\mathbf{1}^{(A)}}} \right\} \equiv s_{\overline{\mathcal{A}_1\mathcal{B}}}^{(A)}$ .

Measurement practice affirms that *Alice* spatiotemporal laser ranging distance  $(t, s)_{\overline{\mathcal{A}_1\mathcal{B}}}$  and her layout of light clocks are equal  $(t, s)_{\overline{\mathcal{A}_1\mathcal{B}}} = (t, s)_{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(A_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}}$ . By counting the *units* and noting the *consecutive or adjacent way of their connection* in her material model (6) *Alice* produces a direct *physical measure* of her **spatiotemporal distance** to *Bob*

$$\begin{aligned} (t, s)_{\overline{\mathcal{A}_1\mathcal{B}}} &\stackrel{(6)}{=} (t, s)_{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(A_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}} \\ &=: \left( t_{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(A_1)}|_{\mathcal{A}}} , s_{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}} \right) \\ &\stackrel{(3)(5)}{=} \left( t_{\overline{\mathcal{A}_1\mathcal{A}}}^{(A)} \cdot t_{\mathbf{1}^{(A)}} , s_{\overline{\mathcal{A}\mathcal{B}}}^{(A)} \cdot s_{\mathbf{1}^{(A)}} \right) = \left( t_{\overline{\mathcal{A}_1\mathcal{A}}} , s_{\overline{\mathcal{A}\mathcal{B}}} \right) \quad . \quad (7) \end{aligned}$$

Finally we express the measurement object and *Alice* constructed sequence of light clocks (her material model) in terms of those measured ratios from her physical measure  $(t, s)_{\overline{\mathcal{A}_1\mathcal{B}}}$

$$\begin{aligned} \overline{\mathcal{A}_1\mathcal{B}} &\stackrel{(6)}{=}_{t,s} \mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \underbrace{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}}}_{\equiv \mathbf{1}_s^{(A_1)}|_{\mathcal{A}}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}} \\ &\stackrel{(3)(5)}{=}_{t,s} \underbrace{t_{\overline{\mathcal{A}_1\mathcal{A}}}^{(A)} \cdot \mathbf{1}_t^{(A)}}_{\overline{\mathcal{A}_1\mathcal{A}}} * \underbrace{s_{\overline{\mathcal{A}\mathcal{B}}}^{(A)} \cdot \mathbf{1}_s^{(A)}}_{\overline{\mathcal{A}\mathcal{B}}} \\ &=: \left( t_{\overline{\mathcal{A}_1\mathcal{A}}}^{(A)} , s_{\overline{\mathcal{A}\mathcal{B}}}^{(A)} \right) \cdot \mathbf{1}^{(A)} \quad . \quad (8) \end{aligned}$$

## 2.6 Indirect laser ranging

For the direct measurement of spatial distances *Alice* successively constructs an adjacent layout of light clocks  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_n)}$ . This material model represents her simultaneous

straight measurement path  $\overline{\mathcal{AB}}$  (see figure 2b). The path of light through the complete stack of mirrors and back  $\mathcal{L}_1 \rightsquigarrow \mathcal{L}_{n+1} \rightsquigarrow \mathcal{L}_1$  is identical with the enveloping light of a layout of individual light clock ticks  $\mathbf{1}^{(\mathcal{A}_i)} : \mathcal{L}_i \rightsquigarrow \mathcal{L}_{i+1} \rightsquigarrow \mathcal{L}_i \dots$  along the way. The length of the stack of light clocks  $\overline{\mathcal{A}_1 \mathcal{A}_n}$  equals the combined length of its individual pieces

$$\begin{aligned} s_{\overline{\mathcal{A}_1 \mathcal{A}_n}} &= s_{\mathbf{1}^{(\mathcal{A}_1)}} + \dots + s_{\mathbf{1}^{(\mathcal{A}_n)}} \\ &= n \cdot s_{\mathbf{1}^{(\mathcal{A})}} \end{aligned}$$

because in *Alice* layout every two adjacent light clocks satisfy  $s_{\mathbf{1}^{(\mathcal{A}_n)}} \cdot s_{\mathbf{1}^{(\mathcal{A}_{n+1})}} \equiv -s_{\mathbf{1}^{(\mathcal{A}_{n+1})}}$  similar to step (c) of her successive construction.

To consecutively and adjacently combine 3 resp. 5 light clocks in her construction steps (b) and (e) we have utilized the *isotropy* of moving light. The universal Light principle also holds for more general connections of identical and comoving light clocks. Therefore in all *local* laser ranging realizations from *Alice* towards *Bob*  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  we find that

- The number of consecutive (light clock) ticks during her laser ranging waiting interval  $\overline{\mathcal{A}_1 \mathcal{A}_2}$  is identical with
- the number of adjacent (ticking) light clocks along her laser ranging route  $\overline{\mathcal{AB}}$

$$t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}^{(\mathcal{A})} \equiv \left\{ \frac{t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}}{t_{\mathbf{1}^{(\mathcal{A})}}} \right\} := \# \left\{ \mathbf{1}_t^{(\mathcal{A})} \right\} \stackrel{!}{=} \# \left\{ \mathbf{1}_s^{(\mathcal{A}_i)} \right\} =: \left\{ \frac{s_{\overline{\mathcal{AB}}}}{s_{\mathbf{1}^{(\mathcal{A})}}} \right\} \equiv s_{\overline{\mathcal{AB}}}^{(\mathcal{A})} . \quad (9)$$

Only thereby meaningful measurement practice with light clocks becomes possible. The successive construction of well defined measurement products is independent from the scale of the light clock (the used measurement unit). The stepwise - locally well defined - laid out *global* straight measurement path  $\mathbf{1}_s^{(\mathcal{A}_1)} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)}$  is therefore universal.

Due to this local regularity (9) *Alice* can characterize her simultaneous measurement path  $\overline{\mathcal{AB}}$  by means of light clock ticks  $\mathbf{1}_t^{(\mathcal{A})}$  along her waiting interval  $\overline{\mathcal{A}_1 \mathcal{A}_2}$  (see figure 3). Locally the set of all simultaneous moments along her *simultaneity line*  $\mathcal{A}_n \in \overline{\mathcal{AB}}$  comes from laser rangings  $\mathcal{A}' \rightsquigarrow \mathcal{A}_n \rightsquigarrow \mathcal{A}''$  where preceding emission and subsequent reception  $\mathcal{A}', \mathcal{A}'' \in \overline{\mathcal{A}_1 \mathcal{A}_2}$  have a symmetrical waiting time  $t_{\overline{\mathcal{A}' \mathcal{A}}} = t_{\overline{\mathcal{A} \mathcal{A}''}}$  relative to *Alice* moment  $\mathcal{A}$ .

In (5) *Alice* measures the radar distance to *Bob*  $s_{\overline{\mathcal{AB}}}$  directly along her (potentially global) simultaneous measurement path  $\overline{\mathcal{AB}}$ . (At least locally) *Alice* can also indirectly determine the length  $s$  of her laser ranging path  $\overline{\mathcal{AB}}$  by means of direct measurement of her round trip duration  $t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}$

$$\begin{aligned} s_{\overline{\mathcal{AB}}} &\stackrel{(5)}{:=} s_{\overline{\mathcal{AB}}}^{(\mathcal{A})} \cdot s_{\mathbf{1}^{(\mathcal{A})}} \\ &\stackrel{(9)}{=} t_{\overline{\mathcal{A}_1 \mathcal{A}_2}}^{(\mathcal{A})} \cdot \underbrace{s_{\mathbf{1}^{(\mathcal{A})}}}_{\stackrel{(1)}{=:} \frac{c}{2} \cdot t_{\mathbf{1}^{(\mathcal{A})}}} \stackrel{(3)}{=:} \frac{c}{2} \cdot t_{\overline{\mathcal{A}_1 \mathcal{A}_2}} . \end{aligned} \quad (10)$$

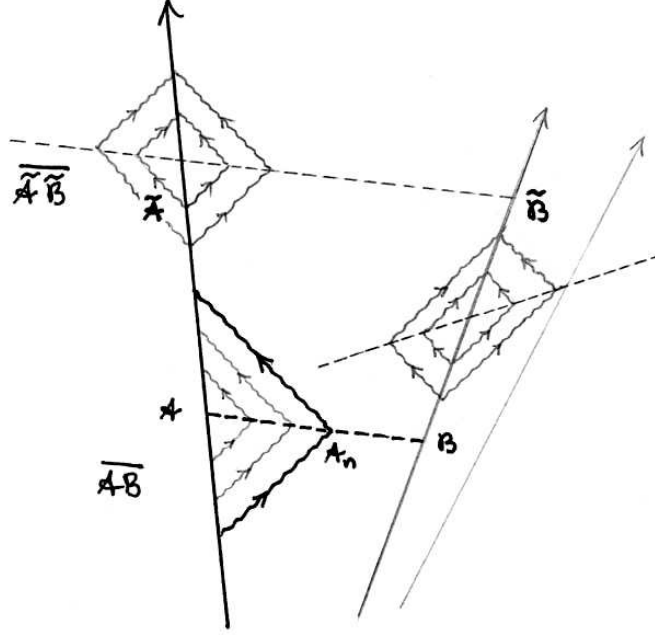


Figure 3: (local) indirect characterization of simultaneous straight measurement paths

That is the familiar measurement principle of laser ranging practice. We *grasp the known* relation *methodologically* from the direct construction of basic physical measures (3) and (5) and their actual light connection (9).

The direct measurement (5) of radar distance  $s_{\overline{AB}}$  is well defined for local as well as for global laser ranging configurations from *Alice* towards *Bob*  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$ . Instead her indirect determination (10) of laser ranging distances  $s_{\overline{AB}}$  by means of measuring round trip times  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}}$  is justified only as long as *Alice* satisfies a measurement condition. She needs to preserve her state of motion during her radar waiting interval  $\overline{\mathcal{A}_1\mathcal{A}_2}$ . After emitting the light pulse in  $\mathcal{A}_1$  *Alice* has to *remain at rest*. She must not distort the indirect measurement by accelerating *out of her own power* away from or towards the returning light pulse from  $\mathcal{B}$ . When we align local clusters of (ticking) light clocks  $\mathbf{1}^{(A)}$  during sufficiently small periods of individual light clock ticks as in (9) this implicit condition is guaranteed. For meaningful indirect measurements in global laser ranging configurations it must be obeyed.

Under that condition we can express the direct physical measure of *Alice* spatiotemporal distance to *Bob* (7) in terms of *Alice* indirect laser ranging measurement  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$

$$(t, s)_{\overline{\mathcal{A}_1\mathcal{B}}} \stackrel{(7)(10)}{=} \left( \frac{1}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}} , \frac{c}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}} \right) \quad (11)$$

and - provided this indirect spatiotemporal physical measure - the measurement object by

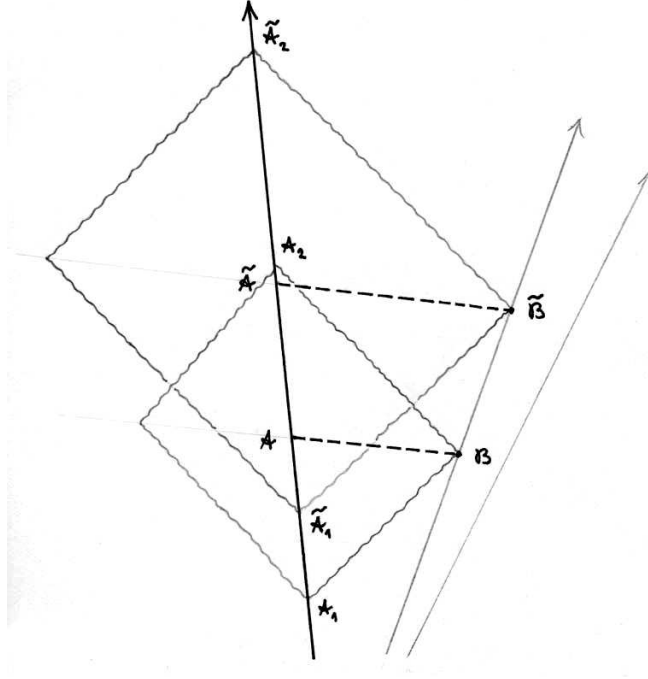


Figure 4: combination of two elementary laser ranging measurements

means of her material model

$$\overline{\mathcal{A}_1 \mathcal{B}}_{(t,s)} \stackrel{(8)(10)}{=} \left( \frac{1}{2} \cdot t_{\mathcal{A}_1 \mathcal{A}_2}^{(\mathcal{A})}, t_{\mathcal{A}_1 \mathcal{A}_2}^{(\mathcal{A})} \right) \cdot \mathbf{1}^{(\mathcal{A})} . \quad (12)$$

## 2.7 Composing elementary laser ranging

During global laser ranging Alice must preserve her state of motion. Measurement object Bob instead can move arbitrarily. With every single laser ranging measurement  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  Alice determines Bob's relative position at the moment  $\mathcal{B}$  when the radar pulse reflects. Alice can determine Bob's motion  $\overline{\mathcal{B}\tilde{\mathcal{B}}}$  from two laser rangings  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  and  $\tilde{\mathcal{A}}_1 \rightsquigarrow \tilde{\mathcal{B}} \rightsquigarrow \tilde{\mathcal{A}}_2$  towards the two consecutive moments  $\mathcal{B}$  and  $\tilde{\mathcal{B}}$  of Bob (see figure 4). Combining those two elementary operations gives Alice general laser ranging measurement

$$\begin{aligned} (t, s)_{\overline{\mathcal{B}\tilde{\mathcal{B}}}} &\stackrel{(6)}{=} (t, s)_{\overline{\mathcal{B}\mathcal{A}} * \overline{\mathcal{A}\tilde{\mathcal{A}}} * \overline{\tilde{\mathcal{A}}\tilde{\mathcal{B}}}} \\ &\stackrel{(7)}{=} \underbrace{(t, s)_{\overline{\mathcal{B}\mathcal{A}}}}_{(0, -s_{\overline{\mathcal{A}\mathcal{B}}})} + \underbrace{(t, s)_{\overline{\mathcal{A}\tilde{\mathcal{A}}}}}_{(t_{\overline{\mathcal{A}\tilde{\mathcal{A}}}}, 0)} + \underbrace{(t, s)_{\overline{\tilde{\mathcal{A}}\tilde{\mathcal{B}}}}}_{(0, s_{\overline{\tilde{\mathcal{A}}\tilde{\mathcal{B}}}})} \\ &= \left( t_{\overline{\mathcal{A}\tilde{\mathcal{A}}}}, s_{\overline{\tilde{\mathcal{A}}\tilde{\mathcal{B}}}} - s_{\overline{\mathcal{A}\mathcal{B}}} \right) . \end{aligned} \quad (13)$$



Thus  $\mathcal{A}$ lice measures a distant segment of  $\mathcal{B}$ ob's motion  $\overline{\mathcal{B}\mathcal{B}}$  by means of

1. *constructing* her straight simultaneous measurement paths towards  $\mathcal{B}$ ob (represented by an adjacently connected swarm of comoving light clocks  $\mathbf{1}_s^{(\mathcal{A}_1)} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)}$  )
2. *enclosing* measurement object  $\overline{\mathcal{B}\mathcal{B}}$  in between her measurement paths  $\overline{\mathcal{A}\mathcal{B}}$  and  $\overline{\tilde{\mathcal{A}}\mathcal{B}}$

$$\overline{\mathcal{B}\mathcal{B}} =_{t,s} \overline{\mathcal{B}\mathcal{A}} * \overline{\mathcal{A}\tilde{\mathcal{A}}} * \overline{\tilde{\mathcal{A}}\mathcal{B}} \quad (14)$$

3. *projecting* measurement object  $\overline{\mathcal{B}\mathcal{B}}$  between resp. along her constructed simultaneity lines for the temporal  $t$  resp. spatial  $s$  component of her physical measure (13).

Without loss of generality we may assume  $\mathcal{A}$ lice and  $\mathcal{B}$ ob to coincide in the initial moment  $\mathcal{P}$ . Now one laser ranging configuration becomes trivial and the other one is  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  (see figure 5). This combination  $\mathcal{P} \rightarrow \mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  provides  $\mathcal{A}$ lice indirect laser ranging physical measure of  $\mathcal{B}$ ob's **motion**

$$(t, s)_{\overline{\mathcal{P}\mathcal{B}}} \stackrel{(13)(11)}{=} \left( t_{\overline{\mathcal{P}\mathcal{A}_1}} + \frac{1}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}} , \frac{c}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}} \right) \quad (15)$$

and - provided this spatiotemporal physical measure - the material model by means of which  $\mathcal{A}$ lice reproduces with her measurement units  $\mathbf{1}^{(\mathcal{A})}$  a segment of the relative motion of  $\mathcal{B}$ ob

$$\overline{\mathcal{P}\mathcal{B}} \stackrel{(12)}{=}_{t,s} \left( t_{\overline{\mathcal{P}\mathcal{A}_1}}^{(\mathcal{A})} + \frac{1}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(\mathcal{A})} , t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(\mathcal{A})} \right) \cdot \mathbf{1}^{(\mathcal{A})} . \quad (16)$$

By consecutive and adjacent connection of light clocks  $\mathbf{1}^{(\mathcal{A})}$  in elementary laser ranging configurations  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$   $\mathcal{A}$ lice physically specifies (direct and indirect) **basic measures** of *duration*  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(\mathcal{A})} \cdot t_{\mathbf{1}^{(\mathcal{A})}}$  (3) and *length*  $s_{\overline{\mathcal{A}\mathcal{B}}}^{(\mathcal{A})} \cdot s_{\mathbf{1}^{(\mathcal{A})}}$  (5). Adjacent and consecutive connection of light clocks are basic measurement operations. From the unity of both aspects in laser ranging practice  $\mathcal{A}$ lice grasps the *unified* physical measure of *spatiotemporal distance*  $(t, s)_{\overline{\mathcal{A}_1\mathcal{B}}}$  (7). Finally from combining two elementary laser ranging configurations  $\mathcal{A}$ lice determines the unified physical measure of  $\mathcal{B}$ ob's *motion*  $(t, s)_{\overline{\mathcal{B}\mathcal{B}}}$  (13).

## 2.8 Measurement termini

Initially  $\mathcal{A}$ lice expressed her work experience with demonstrable objects in relative motion and her practical needs of navigation in simple colloquial language. In a *measurement* she physically specifies properties of measurement object  $\mathcal{O}$ tto by *constructible substitution*. In basic measurements she constructs a material model which (sufficiently precisely) reproduces respective aspects of his real motion. Provided such skills - to quantify  $\mathcal{O}$ tto's motion  $m_{\mathcal{O}}$  -  $\mathcal{A}$ lice can return to the same objects in relative motion and pursue planning her interventions

more reliably. She cultivates the possibility to physically specify familiar notions of space and time by introducing a measurement practice.

Her *measurement principle* is based on:

- Light principle
- classical construction of light clocks  $\mathbf{1}^{(A)}$
- measurement operations of direct and indirect connection of measurement units  $\mathbf{1}^{(A)}$  by means of the independent motion of light.

Alice constructs assemblies of long layouts of (ticking) light clocks  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_n)}$  and enduring sequences of (light clock) ticks  $\mathbf{1}_t^{(A)} * \dots * \mathbf{1}_t^{(A)}$  to reproduce distances and durations of Otto's relative motion. For the construction of those material models Alice always performs the same procedure of operations under the same (measurement) conditions. We designate elements of those constructible substitutions as measurement termini.

In starting snapshot figure 1 we illustrate only observed objects and observers in motion. Provided the construction of measurement units  $\mathbf{1}^{(A)}$  and the convention of rules for their proper utilization we introduce notions - about real physical substitutions - which did not yet exist in the uncultivated beginning. Along figures 2, 3, 4 we define *physical notions* by means of measurement operations with light clocks. Step by step we introduce operational denominations which specify aspects of Alice measurement practice precisely.

All individual aspects are naturally connected in measurement practice. Their common origin inherits a genetic interrelation between measurement termini - about elements of the physical substitution - and between corresponding terms in the mathematical formulation. Each measurement terminus *emphasizes* an aspect of Alice laser ranging practice *without separating* them from their interrelation and without treating them absolute in disregard of all implicit conditions. Their definition from one concrete measurement practice induces a unity of measurement termini. Provided those operational definitions the interrelation of measurement termini becomes transparent. To avoid paradoxes in eventual applications {4} the method of their formation and the conditions have to be considered.

The definition of Alice unit length  $s_{\mathbf{1}^{(A)}}$  emphasizes the length of her light clock without forgetting the rest of the ticking light clock  $\mathbf{1}^{(A)}$ . The utilization of this measurement unit essentially refers to the oscillating light therein. With moving light we always find the length of its motion  $s_{\mathbf{1}^{(A)}}$  (within the (ticking) light clock) together with its duration  $t_{\mathbf{1}^{(A)}}$  (of (light clock) ticks). In measurement practice with light clocks  $\mathbf{1}^{(A)}$  *both moments* 'length' and 'duration' *of motion* are always addressed *unified*.

The definition of Alice straight simultaneous measurement path  $\overline{AB}$  specifies the successive formation of an adjacent layout of comoving light clocks  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_n)}$  towards Bob. A straight time like measurement path of Alice  $\overline{A_1A_2}$  specifies the successive formation of a consecutive sequence of light clocks  $\mathbf{1}_t^{(A)} * \dots * \mathbf{1}_t^{(A)}$  where her light clock remains at

rest.<sup>9</sup> Both material models  $\mathbf{1}_s^{(\mathcal{A}_1)} * \dots * \mathbf{1}_s^{(\mathcal{A}_n)}$  and  $\mathbf{1}_t^{(\mathcal{A})} * \dots * \mathbf{1}_t^{(\mathcal{A})}$  physically specify measurement objects  $\overline{\mathcal{AB}}$  resp.  $\overline{\mathcal{A}_1\mathcal{A}_2}$  with regards to their length  $s_{\overline{\mathcal{AB}}} = s_{\overline{\mathcal{AB}}}^{(\mathcal{A})} \cdot s_{\mathbf{1}^{(\mathcal{A})}}$  resp. duration  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}} = t_{\overline{\mathcal{AB}}}^{(\mathcal{A})} \cdot t_{\mathbf{1}^{(\mathcal{A})}}$ .

In  $\mathcal{A}$ lice elementary laser ranging towards  $\mathcal{B}$ ob  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  we distinguish  $\mathcal{A}$ lice laser ranging path  $\overline{\mathcal{AB}}$  from the length of that route  $s_{\overline{\mathcal{AB}}}$ . Similarly we distinguish  $\mathcal{A}$ lice laser ranging measurement interval  $\overline{\mathcal{A}_1\mathcal{A}_2}$  from the duration of that interval  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}}$ . We distinguish measurement object  $\mathcal{O}_m$  and objective measure  $m_{\mathcal{O}}$  - but we do not separate them. For any measure  $m$  we always adhere to its extensional determination  $\mathcal{O}$ .

## 2.9 Mathematical formulation of basic physical connections

We conceptualize basic measurements as constructible substitutions. In the development of measurement practice we have rigorously distinguished measurement object  $\mathcal{O}_m$  and objective measure  $m_{\mathcal{O}}$ . For the construction of material models measurement units  $\mathbf{1}$  are (firstly manufactured and then) physically connected  $\mathbf{1} * \dots * \mathbf{1}$ . The connected layout (approximately) reproduces the measurement object  $\mathcal{O}$  with regard to its property  $m$ . The objective measure  $m_{\mathcal{O}}$  is - (practically) substitutable by the material model and by this means it is - physically specified.

The *objective knowledge* of that physical substitution is that the connecting operation - when carried out by different individuals under the same conditions - always leads to equal measurement products. The measurements have physical meaning because the operations are universally realizable with respect to the available propagation of light and because they are intersubjectively interchangeable with regard to the individual observer [14]. The technique is preserved until in the empirical practice one oversteps unforeseen conditions, physically specifies them further and thus evolves - in a continual historic process - the measurement practice and its (physically founded) mathematical formulation.<sup>10</sup>

Mathematical addition and physical 'addition' are different operations. In the physical 'addition' real measurement *objects*  $\mathcal{O}_m, \mathcal{R}_m$  - e.g. (Light clock) ticks  $\mathbf{1}_t^{(\mathcal{A})}$ , (ticking) light clocks  $\mathbf{1}_s^{(\mathcal{A})}$ , laser ranging intervals or paths - are physically *connected*

$$\mathcal{O}_m * \mathcal{R}_m \quad .$$

The type of their physical concatenation determines if the connected physical measures  $m_{\mathcal{O}}$  are simply calculated by arithmetic addition or if the resulting measure is formulated by another kind of mathematical operation. The physical connection of measurement objects  $\mathcal{O}_m$  explains the mathematical formulation of connected objective measures  $m_{\mathcal{O}}$ .

By the Light principle the product of a *local* concatenation of light clocks is independent from the connecting order. The connected product  $\mathbf{1}^{(\mathcal{A})} * \dots * \mathbf{1}^{(\mathcal{A})}$  is independent from the *size* of the individual light clocks  $\mathbf{1}^{(\mathcal{A})}$  and from the succession of the consecutive and adjacent

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<sup>9</sup>This measurement condition allows for a well-defined indirect determination of the radar distance  $s_{\overline{\mathcal{AB}}}$  by means of measuring the radar round trip duration  $t_{\overline{\mathcal{A}_1\mathcal{A}_2}}$  in global laser ranging configurations  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$ .

<sup>10</sup>Remembering Feynman's motto: *Yesterdays sensation is todays calibration and tomorrows background.*

way of connecting individual parts, e.g. first  $\mathbf{1}_t^{(A)} * \dots * \mathbf{1}_t^{(A)}$  and then the  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_n)}$  or permutations thereof. Therefore the material model  $\mathbf{1}_t^{(A)} * \dots * \mathbf{1}_t^{(A)} * \dots * \mathbf{1}_s^{(A_n)}$  is well-constructed and universal.<sup>11</sup>

Under these conditions the *bookkeeping of numbers* (of connected units  $\mathbf{1}^{(A)}$ ) - in the assembly of partial material models  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_i)}$ ,  $\mathbf{1}_s^{(A_{i+1})} * \dots * \mathbf{1}_s^{(A_n)}$  and  $\mathbf{1}_t^{(A)} * \dots * \mathbf{1}_t^{(A)}$  to a complete material model  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_i)} * \dots * \mathbf{1}_s^{(A_n)} * \dots * \mathbf{1}_t^{(A)}$  (for basic physical measures in (3), (5) and (7)) - *is allowed* to determine the number (of connected units  $\mathbf{1}^{(A)}$ ) in the whole material model *arithmetically by adding* the number (of connected units  $\mathbf{1}^{(A)}$ ) in its parts [1].

Thus *basic measures* of physically connected objects  $\mathcal{O}_l$ ,  $\mathcal{R}_l$  are arithmetically *added*

$$m_{\mathcal{O}*\mathcal{R}} = m_{\mathcal{O}*}^{(A)} \cdot \mathbf{1}^{(A)} = m_{\mathcal{O}}^{(A)} \cdot \mathbf{1}^{(A)} + m_{\mathcal{R}}^{(A)} \cdot \mathbf{1}^{(A)} = m_{\mathcal{O}} + m_{\mathcal{R}} \quad .$$

The mathematical formulation of connected basic measures  $m_{\mathcal{O}}$  arises from the physical operation of connecting objects  $\mathcal{O}_m$ . In this way we obtained  $s_{\overline{\mathcal{A}_1\mathcal{A}_i}} + s_{\overline{\mathcal{A}_{i+1}\mathcal{A}_n}} = s_{\overline{\mathcal{A}_1\mathcal{A}_n}}$  or  $t_{\overline{\mathcal{A}_1\mathcal{A}}} + t_{\overline{\mathcal{A}\mathcal{A}_2}} = t_{\overline{\mathcal{A}_1\mathcal{A}_2}}$  or unified  $(t, s)_{\overline{\mathcal{A}_1\mathcal{B}}} = (t, s)_{\overline{\mathcal{A}_1\mathcal{A}*}\overline{\mathcal{A}\mathcal{B}}} = (t, s)_{\overline{\mathcal{A}_1\mathcal{A}}} + (t, s)_{\overline{\mathcal{A}\mathcal{B}}} \stackrel{(15)}{=} t_{\overline{\mathcal{A}_1\mathcal{A}}} + c \cdot t_{\overline{\mathcal{A}\mathcal{A}}}$ . The *vectorial character* of corresponding basic spatiotemporal measures follows from the adjacent and consecutive way of connecting light clocks  $\mathbf{1}^{(A)}$ .

On this basis one can understand the physical meaning of assertions on the one-way propagation of light. In elementary laser ranging  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  Alice can determine physical measures for the departing and for the returning light. The measured durations satisfy the following relation

$$t_{\overline{\mathcal{A}_1\mathcal{B}}}^{(A)} \stackrel{(11)}{=} \frac{1}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(A)} \stackrel{(11)}{=} t_{\overline{\mathcal{B}\mathcal{A}_2}}^{(A)} \quad . \quad (17)$$

Alice can not specify if the light pulse travels on the way out  $\mathcal{A}_1 \rightsquigarrow \mathcal{B}$  towards Bob equally or different than on the way back  $\mathcal{B} \rightsquigarrow \mathcal{A}_2$ . Such metrical assertion would only be physically meaningful if corresponding basic measurements could be provided. That is not the case by means of laser ranging with light clocks  $\mathbf{1}^{(A)}$ . Alice never deals with just one light ray; she always operates with both outgoing and returning pulse. Without physical substantiation such relations - about 'hypothetical measures' for the one-way propagation of light - remain mathematically imaginable but physically unjustified. Similarly relation (17) does not address what happens *inside* measurement unit  $\mathbf{1}^{(A)} : \mathcal{L}_I \rightsquigarrow \mathcal{L}_{II} \rightsquigarrow \mathcal{L}_I \dots$  (when light travels between separated parts to the right  $\mathcal{L}_I \rightsquigarrow \mathcal{L}_{II}$  vs. to the left  $\mathcal{L}_{II} \rightsquigarrow \mathcal{L}_I$ ).

The meaning of basic physical measures arises - not by chopping measurement units  $\mathbf{1}^{(A)}$  into pieces but instead - by *connecting many* measurement units  $\mathbf{1}^{(A)}$  (each taken as an

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<sup>11</sup>In a similar way we have already explained the universality of the intrinsically straight layout of comoving identical light clocks  $\mathbf{1}_t^{(A)}|_{\mathcal{A}} * \dots * \mathbf{1}_t^{(A)}|_{\mathcal{B}}$  in (9). Alice constructs this layout by pure adjacent connection of light clocks. In elementary laser ranging  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  it represents her simultaneous straight measurement path towards Bob  $\overline{\mathcal{A}\mathcal{B}}$ .

inseparable unity) *to construct measurement means*  $\mathbf{1}^{(A)} * \dots * \mathbf{1}^{(A)}$ . The physical meaning of relations between basic physical measures is how to *externally assemble* measurement units  $\mathbf{1}^{(A)}$  for the construction of material models e.g. in  $\mathbf{1}_t^{(A)} * \dots * \mathbf{1}_t^{(A)}$  or in  $\mathbf{1}_s^{(A_1)} * \dots * \mathbf{1}_s^{(A_n)}$  or connected in  $\mathbf{1}_t^{(A)} * \dots * \underbrace{\mathbf{1}_t^{(A)}}_{\equiv \mathbf{1}_s^{(A_1)}} * \dots * \mathbf{1}_s^{(A_n)}$ .

In laser ranging practice we solely deal with two-way light cycles (i) inside individual light clocks  $\mathbf{1}^{(A)}$  and (ii) in suitably connected configurations of light clocks. Within a light clock  $\mathbf{1}^{(A)} : \mathcal{L}_I|_{\mathcal{A}} \rightsquigarrow \mathcal{L}_{II} \rightsquigarrow \mathcal{L}_I \dots$  we assume by the isotropy of light that beginning at moment  $\mathcal{A}$  a *two-way* light cycle to the left travels the same unit distance  $s_{\mathbf{1}^{(A)}}$  to the turning point ' $\mathcal{A} - s_{\mathbf{1}^{(A)}}$ ' left of moment  $\mathcal{A}$  as the other *two-way* light cycle travels to his turning point ' $\mathcal{A} + s_{\mathbf{1}^{(A)}}$ ' right of moment  $\mathcal{A}$  (as we had used in steps (c) and (e) in the construction of straight measurement paths). The measurement principle is that: Given the *elementary* two-way light cycle in the measurement unit  $\mathbf{1}^{(A)}$  we analyze in measurement practice complex configurations of those elementary two-way light cycles in suitably constructed layouts of ticking light clocks  $\mathbf{1}^{(A)}$ . Intrinsically  $\mathcal{A}$ lice can only connect neighboring light clocks in two meaningful ways: adjacently and consecutively.

Therefore also the one-way-motion of light is physically specified by means of suitable configurations of two-way light cycles.  $\mathcal{A}$ lice can construct a measurement layout of light clocks (6)

$$\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \underbrace{\mathbf{1}_t^{(A_1)}|_{\mathcal{A}}}_{\equiv \mathbf{1}_s^{(A_1)}|_{\mathcal{A}}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}$$

which substitutes laser ranging configuration  $\mathcal{A}_1 \rightsquigarrow \mathcal{B} \rightsquigarrow \mathcal{A}_2$  and from which she can directly determine her basic physical measures for both one-way pulses of light

$$(t, s)_{\overline{\mathcal{A}_1 \mathcal{B}}} \stackrel{(15)}{=} \frac{1}{2} \cdot \overline{t_{\mathcal{A}_1 \mathcal{A}_2}} + \frac{c}{2} \cdot \overline{t_{\mathcal{A}_1 \mathcal{A}_2}} \quad \text{and} \quad (18)$$

$$(t, s)_{\overline{\mathcal{B} \mathcal{A}_2}} \stackrel{(15)}{=} \frac{1}{2} \cdot \overline{t_{\mathcal{A}_1 \mathcal{A}_2}} - \frac{c}{2} \cdot \overline{t_{\mathcal{A}_1 \mathcal{A}_2}} \quad . \quad (19)$$

Here all *arithmetic operations* between objective measures  $m_{\mathcal{O}}$  " + ", " - ", "  $\frac{1}{2}$  ." etc. originate from  $\mathcal{A}$ lice elementary connecting operations on corresponding measurement objects  $\mathcal{O}_m$ ; more precisely on light clocks, i.e. her measurement units  $\mathbf{1}_m$

1. The **operational meaning** of relation (18) is that starting from moment  $\mathcal{A}_1$  after half of her laser ranging interval  $\overline{\mathcal{A}_1 \mathcal{A}}|_{\mathcal{A}_1} := \frac{1}{2} \cdot \overline{t_{\mathcal{A}_1 \mathcal{A}_2}} \cdot \mathbf{1}_t^{(A)}$  in moment  $\mathcal{A}$   $\mathcal{A}$ lice successively constructs a straight instantaneous measurement path  $\overline{\mathcal{A} \mathcal{B}}|_{\mathcal{A}} := \overline{t_{\mathcal{A}_1 \mathcal{A}_2}} \cdot \mathbf{1}_s^{(A)}$  to  $\mathcal{B}$ ob. *Adjacent connection* of  $\mathbf{1}_s^{(A_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_s^{(A_n)}|_{\mathcal{B}}$  to consecutive layout  $\mathbf{1}_t^{(A_1)}|_{\mathcal{A}_1} * \dots * \mathbf{1}_t^{(A_1)}|_{\mathcal{A}}$  occurs in light clock  $\mathbf{1}_t^{(A_1)}|_{\mathcal{A}} \equiv \mathbf{1}_s^{(A_1)}|_{\mathcal{A}}$  at moment  $\mathcal{A}$  after half of  $\mathcal{A}$ lice laser ranging waiting interval  $\overline{\mathcal{A}_1 \mathcal{A}_2} \equiv \overline{\mathcal{A}_1 \mathcal{A}} * \overline{\mathcal{A} \mathcal{A}_2}$ .
2. Similarly the operational meaning of relation (19) is that returning from moment  $\mathcal{B}$  along all of her measurement path  $\overline{\mathcal{B} \mathcal{A}}|_{\mathcal{B}} := -\overline{t_{\mathcal{A}_1 \mathcal{A}_2}} \cdot \mathbf{1}_s^{(A)}$  in moment  $\mathcal{A}$   $\mathcal{A}$ lice lays out

a consecutive sequence of light clocks  $\overline{\mathcal{A}\mathcal{A}_2}|_{\mathcal{A}} := \frac{1}{2} \cdot t_{\mathcal{A}_1\mathcal{A}_2}^{(\mathcal{A})} \cdot \mathbf{1}_t^{(\mathcal{A})}$  to reach moment  $\mathcal{A}_2$ . *Consecutive connection* of  $\mathbf{1}_t^{(\mathcal{A}_1)}|_{\mathcal{A}} * \dots * \mathbf{1}_t^{(\mathcal{A}_n)}|_{\mathcal{A}_2}$  to adjacent layout  $\mathbf{1}_s^{(\mathcal{A}_n)}|_{\mathcal{B}} * \dots * \mathbf{1}_s^{(\mathcal{A}_1)}|_{\mathcal{A}}$  again occurs in light clock  $\mathbf{1}_s^{(\mathcal{A}_1)}|_{\mathcal{A}} \equiv \mathbf{1}_t^{(\mathcal{A}_1)}|_{\mathcal{A}}$  after waiting half of her laser ranging interval  $\overline{\mathcal{A}_1\mathcal{A}_2} \equiv \overline{\mathcal{A}_1\mathcal{A}} * \overline{\mathcal{A}\mathcal{A}_2}$ .

### 3 Interrelation of basic spatiotemporal measures

We have specified - the definition and interrelation of measurement termini in - laser ranging practice e.g. from *Alice* towards *Otto*  $\mathcal{P} \rightarrow \mathcal{A}_1 \rightsquigarrow \mathcal{O} \rightsquigarrow \mathcal{A}_3$ . Now we consider one more intrinsic observer *Bob* who measures the same segment  $\overline{\mathcal{P}\mathcal{O}}$  of *Otto's* motion (see figure 5). *Bob* sets up his laser ranging  $\mathcal{P} \rightarrow \mathcal{B}_1 \rightsquigarrow \mathcal{O} \rightsquigarrow \mathcal{B}_2$  in the same way as *Alice*. Following protophysical manufacturing guidelines he can build his own light clock  $\mathbf{1}^{(\mathcal{B})}$ . By means of operating with his own measurement unit he determines *Otto's* relative motion. Step by step *Bob* develops analogous measurement termini as introduced in {2}.

First *Bob constructs* his (dotted) straight simultaneous measurement path towards *Otto*  $\overline{\mathcal{B}\mathcal{O}}$ . Directly by adjacently connecting a swarm of comoving light clocks  $\mathbf{1}_s^{(\mathcal{B}_1)} * \dots * \mathbf{1}_s^{(\mathcal{B}_n)}$  or indirectly by analyzing round trip light signals with his light clock *Bob* can lay out his own measurement paths towards *Otto*  $\overline{\mathcal{B}\mathcal{O}}$  (or back towards *Alice*  $\overline{\mathcal{B}\mathcal{A}}$ ). *Alice* and *Bob* set up their laser ranging technique with the *same measurement principle*: (i) independent propagation of light (ii) intrinsic construction of their respective light clock and (iii) laying out their light clocks in a consecutive and adjacent way. Though the outcomes of their intrinsic measurements are not identical: e.g. *Alice* constructed simultaneity lines  $\overline{\mathcal{A}\mathcal{B}}$ ,  $\overline{\mathcal{A}\mathcal{O}}$  and *Bob's* constructed simultaneity lines  $\overline{\mathcal{B}\mathcal{A}}$ ,  $\overline{\mathcal{B}\mathcal{O}}$  differ as illustrated in figure 3.

(Just as *Alice*) *Bob encloses* and *projects* measurement object *Otto*  $\overline{\mathcal{P}\mathcal{O}}$  in between his constructed simultaneity lines  $\overline{\mathcal{B}\mathcal{O}}$

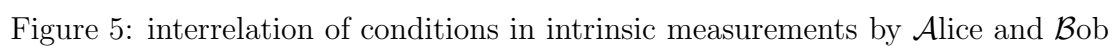
$$\overline{\mathcal{P}\mathcal{O}} \stackrel{(14)}{=} \overline{\mathcal{P}\mathcal{B}} * \overline{\mathcal{B}\mathcal{O}}$$

and finally determines the physical measure of *Otto's* relative motion

$$\begin{aligned} (t, s)_{\overline{\mathcal{P}\mathcal{O}}} &= (t, s)_{\overline{\mathcal{P}\mathcal{B}_1} * \overline{\mathcal{B}_1\mathcal{B}} * \overline{\mathcal{B}\mathcal{O}}} \\ &\stackrel{(13)}{=} \left( t_{\overline{\mathcal{P}\mathcal{B}_1}} + t_{\overline{\mathcal{B}_1\mathcal{B}}} , s_{\overline{\mathcal{B}\mathcal{O}}} \right) . \end{aligned} \quad (20)$$

The intrinsic measurement principle for *Alice* and *Bob* is the same. Both reproduce by consecutive and adjacent connections of their light clocks  $\mathbf{1}^{(\mathcal{A})}$  resp.  $\mathbf{1}^{(\mathcal{B})}$  the same segment of *Otto's* motion

$$\begin{aligned} \overline{\mathcal{P}\mathcal{O}} &= \left( t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{A})} , s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{A})} \right) \cdot \mathbf{1}^{(\mathcal{A})} \\ &= \left( t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})} , s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})} \right) \cdot \mathbf{1}^{(\mathcal{B})} . \end{aligned}$$



Furthermore both can measure the same segment of  $\mathcal{B}$ ob's and of  $\mathcal{A}$ lice' motion

$$\begin{aligned}
\overline{\mathcal{P}\mathcal{B}_1} &= \left( t_{\overline{\mathcal{P}\mathcal{B}_1}}^{(\mathcal{A})}, s_{\overline{\mathcal{P}\mathcal{B}_1}}^{(\mathcal{A})} \right) \cdot \mathbf{1}^{(\mathcal{A})} \\
&= \left( t_{\overline{\mathcal{P}\mathcal{B}_1}}^{(\mathcal{B})}, 0 \right) \cdot \mathbf{1}^{(\mathcal{B})} \\
\overline{\mathcal{P}\mathcal{A}_1} &= \left( t_{\overline{\mathcal{P}\mathcal{A}_1}}^{(\mathcal{A})}, 0 \right) \cdot \mathbf{1}^{(\mathcal{A})} \\
&= \left( t_{\overline{\mathcal{P}\mathcal{A}_1}}^{(\mathcal{B})}, s_{\overline{\mathcal{P}\mathcal{A}_1}}^{(\mathcal{B})} \right) \cdot \mathbf{1}^{(\mathcal{B})} .
\end{aligned}$$

The same segment of  $\mathcal{O}$ tto's motion  $\overline{\mathcal{P}\mathcal{O}}$  is physically specified by  $\mathcal{A}$ lice with respect to light clock  $\mathbf{1}^{(\mathcal{A})}$  and by  $\mathcal{B}$ ob with respect to light clock  $\mathbf{1}^{(\mathcal{B})}$ . The interrelation of light rays in their laser ranging processes is depicted in figure 5.

From their same measurement principle and the interrelation of measurement conditions we can derive the relation between both measurements of the same measurement object  $\overline{\mathcal{P}\mathcal{O}}$

$$\left( t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{A})}, s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{A})} \right) \leftrightarrow \left( t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})}, s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})} \right) .$$

The transformation between  $\mathcal{A}$ lice and  $\mathcal{B}$ ob's measurement results of  $\mathcal{O}$ tto's motion  $\overline{\mathcal{P}\mathcal{O}}$  follows - provided measurements of their own motion  $\overline{\mathcal{P}\mathcal{A}_1}, \overline{\mathcal{P}\mathcal{B}_1}$  - by successive substitution in three steps:

$$\begin{array}{ll}
\text{I} & \overbrace{\left( t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})}, s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})} \right)}^{\overline{\mathcal{P}\mathcal{O}} \text{ measured by } \mathcal{B}\text{ob}} \quad (M, N) \\
\text{II} & (M, N) \quad (A, B, C) \\
\text{III} & (A, B, C) \quad \underbrace{\left( \left( t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{A})}, s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{A})} \right) \& \left( t_{\overline{\mathcal{P}\mathcal{B}_1}}^{(\mathcal{A})}, s_{\overline{\mathcal{P}\mathcal{B}_1}}^{(\mathcal{A})} \right) \right)}_{\overline{\mathcal{P}\mathcal{O}} \& \overline{\mathcal{P}\mathcal{B}_1} \text{ measured by } \mathcal{A}\text{lice}}
\end{array}$$

where we use the following abbreviations for indirect laser ranging measurements by  $\mathcal{A}$ lice

$$A := t_{\overline{\mathcal{P}\mathcal{A}_1}}^{(\mathcal{A})} \quad B := \frac{1}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_2}}^{(\mathcal{A})} \quad C := \frac{1}{2} \cdot t_{\overline{\mathcal{A}_1\mathcal{A}_3}}^{(\mathcal{A})}$$

and by  $\mathcal{B}$ ob

$$M := t_{\overline{\mathcal{P}\mathcal{B}_1}}^{(\mathcal{B})} \quad N := \frac{1}{2} \cdot t_{\overline{\mathcal{B}_1\mathcal{B}_2}}^{(\mathcal{B})} .$$

In **step I** we express  $\mathcal{B}$ ob's indirect physical measure of  $\mathcal{O}$ tto's motion  $(t, s)_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})}$  in terms of  $\mathcal{B}$ ob's direct measurements of laser ranging round-trip durations  $M, N$

$$t_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})} \stackrel{(15)}{=} M + N \tag{21}$$

$$s_{\overline{\mathcal{P}\mathcal{O}}}^{(\mathcal{B})} \stackrel{(15)}{=} c \cdot N . \tag{22}$$



In **step II** we express **Bob's** direct measurements of round-trip durations  $M, N$  in terms of **Alice's** direct duration measurements  $A, B, C$ . In order to substitute the two physical measures  $M, N$  in terms of the physical measures  $A, B, C$  we need two relations between those corresponding time measurements.

The conditions for laser ranging processes by **Bob** and **Alice** are interrelated as depicted in figure 5. The two outgoing light rays  $\overline{\mathcal{A}_1\mathcal{B}_1}$  and  $\overline{\mathcal{A}_1\mathcal{O}}$  partially coincide and the two returning light rays  $\overline{\mathcal{B}_1\mathcal{A}_2}$  and  $\overline{\mathcal{B}_2\mathcal{A}_3}$  are parallel  $\{2.3\}$ . Both triangles  $\mathcal{PB}_1\mathcal{A}_2$  and  $\mathcal{PB}_2\mathcal{A}_3$  are similar. Their sides - along congruent paths  $\overline{\mathcal{PA}}$  resp.  $\overline{\mathcal{PB}}$  and the two light rays - are parallel. Thus we get one relation

$$\frac{M}{A + B + B} = \frac{M + N + N}{A + C + C} \quad . \quad (23)$$

A second relation we get from analyzing the two triangles  $\mathcal{PA}_1\mathcal{B}_1$  and  $\mathcal{PB}_1\mathcal{A}_2$ . According to the *relativity principle* they are intrinsically similar. **Alice** and **Bob** have no way to specify absolute motion. By means of intrinsic measurements they see each others relative motion equivalent. We can understand both triangles  $\mathcal{PA}_1\mathcal{B}_1$  and  $\mathcal{PB}_1\mathcal{A}_2$  as a calibration procedure by means of which **Alice** and **Bob** can compare their light clocks  $\mathbf{1}^{(A)}$  and  $\mathbf{1}^{(B)}$ :

- In  $\mathcal{PA}_1\mathcal{B}_1$  **Alice** sends out two light signals along  $\overline{\mathcal{PA}_1} = A \cdot \mathbf{1}_t^{(A)}$  - the first at moment  $P$  and the second at moment  $\mathcal{A}_1$  after  $\sharp A$  ticks of her light clock - which are received by **Bob** along  $\overline{\mathcal{PB}_1} = M \cdot \mathbf{1}_t^{(B)}$  - the first at moment  $P$  and the second at moment  $\mathcal{B}_1$  after  $\sharp M$  ticks of his light clock.
- In  $\mathcal{PB}_1\mathcal{A}_2$  **Bob** sends out two light signals along  $\overline{\mathcal{PB}_1} = M \cdot \mathbf{1}_t^{(B)}$  - the first at moment  $P$  and the second at moment  $\mathcal{B}_1$  after  $\sharp M$  ticks of his light clock - which are received by **Alice** along  $\overline{\mathcal{PA}_2} = (A + B + B) \cdot \mathbf{1}_t^{(A)}$  - the first at moment  $P$  and the second at moment  $\mathcal{A}_2$  after  $\sharp(A + B + B)$  ticks of her light clock.

If **Alice** and **Bob** use the same measurement unit (i.e. they use physically identical rigid bodies to build their light clocks) then we require that both encounter the same dilation effect for each others relative motion. By the Light principle and by the *symmetry* of the configuration both determine the same ratio between the two durations for receiving both signals (heard from the other) and the duration of the sending interval (measured by themselves)

$$\frac{M}{A} \stackrel{!}{=} \frac{A + B + B}{M} \quad . \quad (24)$$

In **step III** we express **Alice's** direct duration measurements  $A, B, C$  in terms of **Alice's**

indirect laser ranging measures (15) of  $\mathcal{O}$ tto's motion  $(t, s)_{\overline{\mathcal{PO}}}^{(\mathcal{A})}$  and of  $\mathcal{B}$ ob's motion  $(t, s)_{\overline{\mathcal{PB}_1}}^{(\mathcal{A})}$

$$A = t_{\overline{\mathcal{PO}}}^{(\mathcal{A})} - \frac{1}{c} \cdot s_{\overline{\mathcal{PO}}}^{(\mathcal{A})} \quad (25)$$

$$= t_{\overline{\mathcal{PB}}}^{(\mathcal{A})} - \frac{1}{c} \cdot s_{\overline{\mathcal{PB}}}^{(\mathcal{A})} \quad (26)$$

$$B = \frac{1}{c} \cdot s_{\overline{\mathcal{PB}}}^{(\mathcal{A})} \quad (27)$$

$$C = \frac{1}{c} \cdot s_{\overline{\mathcal{PO}}}^{(\mathcal{A})} . \quad (28)$$

After successive insertion of these three steps (see appendix A) we can finally express  $\mathcal{B}$ ob's physical measure of  $\mathcal{O}$ tto's motion  $(t, s)_{\overline{\mathcal{PO}}}^{(\mathcal{B})}$  in terms of  $\mathcal{A}$ lice two measurements of  $\mathcal{O}$ tto's motion  $(t, s)_{\overline{\mathcal{PO}}}^{(\mathcal{A})}$  and of the relative motion of  $\mathcal{B}$ ob  $(t, s)_{\overline{\mathcal{PB}_1}}^{(\mathcal{A})}$

$$\begin{aligned} t_{\overline{\mathcal{PO}}}^{(\mathcal{B})} &\stackrel{(38)}{=} \frac{1}{\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}} \cdot t_{\overline{\mathcal{PO}}}^{(\mathcal{A})} - \frac{1}{\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}} \cdot \frac{v_{\mathcal{B}}}{c^2} \cdot s_{\overline{\mathcal{PO}}}^{(\mathcal{A})} \\ s_{\overline{\mathcal{PO}}}^{(\mathcal{B})} &\stackrel{(37)}{=} - \frac{1}{\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}} \cdot v_{\mathcal{B}} \cdot t_{\overline{\mathcal{PO}}}^{(\mathcal{A})} + \frac{1}{\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}} \cdot s_{\overline{\mathcal{PO}}}^{(\mathcal{A})} \end{aligned}$$

where  $\mathcal{A}$ lice has determined the velocity of the relative motion of  $\mathcal{B}$ ob  $v_{\mathcal{B}} := s_{\overline{\mathcal{PB}}}^{(\mathcal{A})} / t_{\overline{\mathcal{PB}}}^{(\mathcal{A})}$ . With more abbreviations introduced  $\beta := v_{\mathcal{B}}/c$  and  $\gamma := 1/\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}$  we obtain the matrix notation

$$\begin{pmatrix} t_{\overline{\mathcal{PO}}}^{(\mathcal{B})} \\ s_{\overline{\mathcal{PO}}}^{(\mathcal{B})} \end{pmatrix} = \gamma \cdot \begin{pmatrix} 1 & -\beta \cdot \frac{1}{c} \\ -\beta \cdot c & 1 \end{pmatrix} \begin{pmatrix} t_{\overline{\mathcal{PO}}}^{(\mathcal{A})} \\ s_{\overline{\mathcal{PO}}}^{(\mathcal{A})} \end{pmatrix} . \quad (29)$$

This mathematical relation is familiar **Lorentz transformation**  $\Lambda_{\mathcal{AB}} : (t, s)^{(\mathcal{A})} \mapsto (t, s)^{(\mathcal{B})}$ . We define this map in a commutative diagram by three successive pull backs through - the principles of concrete measurements in - the real world (see figure 6). We *grasp the known relation methodologically* from  $\mathcal{A}$ lice and  $\mathcal{B}$ ob's direct construction of basic physical measures of  $\mathcal{O}$ tto's relative motion (step I and III) and from the interrelation of their laser ranging measurement conditions (step II).

While the formalism begins with the *definition* of abstract vectors and with the *postulate* of local Lorentz symmetry we zoom into the methodological basis for those mathematical relations. In a spacetime vector  $\left(t_{\overline{\mathcal{PO}}}^{(\mathcal{A})}, s_{\overline{\mathcal{PO}}}^{(\mathcal{A})}\right)$  each mathematical term is an abbreviation. They are mathematical formulations for measurement termini. In reality each specifies a physical substitution  $t_{\overline{\mathcal{PO}}}^{(\mathcal{A})} := \left\{ \frac{t_{\overline{\mathcal{PO}}}}{t_{1^{(\mathcal{A})}}} \right\}$  and  $s_{\overline{\mathcal{PO}}}^{(\mathcal{A})} := \left\{ \frac{s_{\overline{\mathcal{PO}}}}{s_{1^{(\mathcal{A})}}} \right\}$ . The real 'duration' and 'length'



## 4 Twin paradox

We consider the relative motion between *Alice* and *Bob*. The interrelation of their physical measures is a special case of the general Lorentz transformation  $\Lambda_{AB} : (t, s)^{(A)} \mapsto (t, s)^{(B)}$ . For *Alice* and *Bob* we identify measurement object *Otto* with *Bob*. We assume that *Alice* has physically specified the motion of *Bob*  $\mathcal{O} \equiv \mathcal{B} = \left( t_B^{(A)}, s_B^{(A)} \right) \cdot \mathbf{1}^{(A)}$ . Separate transformation of both components

$$\begin{aligned}
 t_B^{(B)} &\stackrel{(38)}{=} \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot t_B^{(A)} - \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot \frac{v_B}{c^2} \cdot s_B^{(A)} \\
 &= \frac{t_B^{(A)}}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot \left( 1 - \frac{v_B}{c^2} \cdot \frac{s_B^{(A)}}{t_B^{(A)}} \right) = \sqrt{1 - \frac{v_B^2}{c^2}} \cdot t_B^{(A)} \\
 s_B^{(B)} &\stackrel{(37)}{=} - \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot v_B \cdot t_B^{(A)} + \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot s_B^{(A)} \equiv 0
 \end{aligned} \tag{30}$$

gives the physical measure of *Bob* first specified by *Alice* and then by *Bob* himself

$$\begin{aligned}
 \mathcal{B} &= \left( t_B^{(A)}, s_B^{(A)} \right) \cdot \mathbf{1}^{(A)} \\
 &\stackrel{(30)}{=} \left( \underbrace{\sqrt{1 - \frac{v_B^2}{c^2}} \cdot t_B^{(A)}}_{=t_B^{(B)}}, 0 \right) \cdot \mathbf{1}^{(B)} .
 \end{aligned} \tag{31}$$

*Alice* spends more time watching *Bob* travel  $t_B^{(A)}$  than the ride takes for *Bob* himself  $t_B^{(B)}$

$$t_B^{(B)} \stackrel{(31)}{<} t_B^{(A)} .$$

*Alice* observes that her own clock ticks faster then the moving clock of *Bob*.

If reversely we assume that *Bob* has measured the motion of *Alice*  $\mathcal{A} = \left( t_A^{(B)}, s_A^{(B)} \right) \cdot \mathbf{1}^{(B)}$  we obtain similarly the physical measure of *Alice* motion as specified directly by herself

$$\mathcal{A} \stackrel{(30)}{=} \left( \underbrace{\sqrt{1 - \frac{v_B^2}{c^2}} \cdot t_A^{(B)}}_{=t_A^{(A)}}, 0 \right) \cdot \mathbf{1}^{(A)} . \tag{32}$$

*Bob* spends more time watching *Alice* travel  $t_A^{(B)}$  than the ride takes for *Alice* herself  $t_A^{(A)}$

$$t_A^{(A)} \stackrel{(32)}{<} t_A^{(B)} .$$

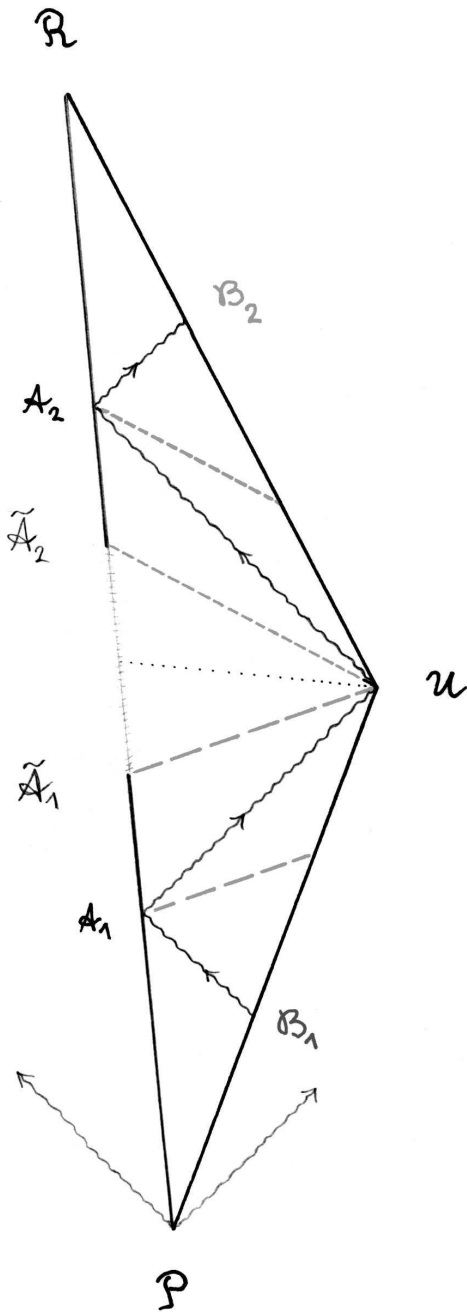


Figure 7: connection for laser ranging in Twin configuration of Alice and Bob<sub>1,2</sub>

$\mathcal{B}$ ob observes that his own clock ticks faster than the moving clock of  $\mathcal{A}$ lice. By the symmetry of their relative motion  $v_{\mathcal{A}}^{(\mathcal{B})} = -v_{\mathcal{B}}^{(\mathcal{A})}$  both  $\mathcal{A}$ lice and  $\mathcal{B}$ ob see the moving clock of the other run slower than their own.

In the Twin configuration  $\mathcal{A}$ lice and  $\mathcal{B}$ ob explore their mutual time dilation in a round trip experiment. They depart at moment  $\mathcal{P}$ . While  $\mathcal{A}$ lice remains at rest  $\mathcal{B}$ ob rides with uniform motion  $v_{\mathcal{B}}^{(\mathcal{A})}$  to a distant turning point  $\mathcal{U}$  and returns with same velocity  $-v_{\mathcal{B}}^{(\mathcal{A})}$  to reunite with  $\mathcal{A}$ lice in future moment  $\mathcal{R}$  (see figure 7).

Throughout the whole round trip  $\mathcal{A}$ lice can observe  $\mathcal{B}$ ob. And vice versa  $\mathcal{B}$ ob will receive all the light signals which  $\mathcal{A}$ lice sends after him from departure until return. If we *assume* that  $\mathcal{A}$ lice observation period of  $\mathcal{B}$ ob's journey  $t_{\mathcal{B}}^{(\mathcal{A})}$  coincides with her own waiting time  $t_{\mathcal{A}}^{(\mathcal{A})}$

$$t_{\mathcal{B}}^{(\mathcal{A})} \equiv t_{\mathcal{A}}^{(\mathcal{A})} \quad (33)$$

then  $\mathcal{B}$ ob spends less time on tour  $t_{\mathcal{B}}^{(\mathcal{B})} \stackrel{(31)(33)}{=} \underbrace{\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}}_{<1} \cdot t_{\mathcal{A}}^{(\mathcal{A})}$  then  $\mathcal{A}$ lice in her waiting line.

If however we *assume* that the period during which  $\mathcal{B}$ ob observes  $\mathcal{A}$ lice  $t_{\mathcal{A}}^{(\mathcal{B})}$  coincides with his own travel time  $t_{\mathcal{B}}^{(\mathcal{B})}$

$$t_{\mathcal{A}}^{(\mathcal{B})} \equiv t_{\mathcal{B}}^{(\mathcal{B})} \quad (34)$$

then  $\mathcal{A}$ lice has spend less time waiting  $t_{\mathcal{A}}^{(\mathcal{A})} \stackrel{(32)(34)}{=} \underbrace{\sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}}}_{<1} \cdot t_{\mathcal{B}}^{(\mathcal{B})}$  then  $\mathcal{B}$ ob on his round

trip.

The combined *conclusion* is a contradiction  $t_{\mathcal{B}}^{(\mathcal{B})} < t_{\mathcal{A}}^{(\mathcal{A})} < t_{\mathcal{B}}^{(\mathcal{B})}$ ; the so called *Twin paradox*.

Both assumptions about  $\mathcal{A}$ lice and  $\mathcal{B}$ ob's mutual observation periods (33) and (34) are perfectly correct; though irrelevant for the Lorentz transformation (29) in the first place. This formula on the interrelation of physical measures does not refer to durations of their mutual observations; it refers to durations of their measurements. To assure a physically meaningful application of mathematical formalisms we can remember that the formation of basic physical measures  $(s, t)$  originates from a constructible substitution. In calculations with the Lorentz formula on the interrelation of physical measures - i.e. between results of measurement operations - we check if implicit conditions for forming these supposed physical measures are satisfied in the first place! In this operationally completed view we can grasp physical limitations of formal calculations in the Lorentz formalism and we understand the physical explanation of the apparent Twin paradox.

We strictly distinguish observation times and measurement times.  $\mathcal{A}$ lice can observe and measure  $\mathcal{B}$ ob throughout the whole trip. From  $\mathcal{B}$ ob's perspective this is not the case anymore. He continues receiving all light signals  $\mathcal{A}$ lice sends after him. Though the middle segment of  $\mathcal{A}$ lice motion  $\widetilde{\mathcal{A}}_1 \widetilde{\mathcal{A}}_2$  is observable but not measurable for him. That is not the fault of

his *measurement object* Alice, nor the fault of his *measurement instruments*; the problem is that the *operation of physical connection* is not realizable. Alice remains waiting in her same state of motion. Bob's light clocks  $\mathbf{1}^{(B_1)}$  and  $\mathbf{1}^{(B_2)}$  remain constructible and function properly on the way out and back. Though they cannot anymore be connected properly for the physical specification of larger distances due to Bob's abrupt change of motion at his  $\mathcal{U}$ -turn point.

The moment  $\mathcal{A}_1$  corresponds to Bob<sub>1</sub>'s last indirect laser ranging  $\mathcal{B}_1 \rightsquigarrow \mathcal{A}_1 \rightsquigarrow \mathcal{B}'_1$  such that the reflected light pulse - awaited to analyze round trip times - reaches Bob<sub>1</sub> before he changes his state of motion Bob<sub>2</sub> at moment  $\mathcal{U}$  - in violation of the measurement condition for indirect laser ranging {2.6}. The moment  $\mathcal{A}'_1$  corresponds to Bob<sub>1</sub>'s last direct laser ranging by means of consecutive and adjacent connection of his light clocks. This is his last directly physically specifiable moment of Alice before in any later construction his individual light clocks  $\mathbf{1}^{(B)}$  have to be assembled completely different next to one another; namely to form a straight simultaneous measurement path with respect to his new state of motion Bob<sub>2</sub>. Similarly Bob<sub>2</sub> can only realize direct and indirect laser ranging measurements of Alice from the moment  $\mathcal{A}'_2$  resp.  $\mathcal{A}_2$  up to the point of  $\overline{\mathcal{R} \text{ return}}$ . Bob<sub>1</sub> and Bob<sub>2</sub> have no possibility to physically specify the segment of Alice motion  $\overline{\widetilde{\mathcal{A}}_1 \widetilde{\mathcal{A}}_2}$  by use of their light clocks, i.e. by means of consecutive and adjacent connection and in compliance with measurement conditions.

During his complete round trip Bob measures two segments of Alice relative motion  $\overline{\mathcal{P} \widetilde{\mathcal{A}}_1}$  and  $\widetilde{\mathcal{A}}_2 \mathcal{R}$ . By the symmetry of their relative motion Bob observes all processes for moving Alice run slower. Like from the opposite perspective for Alice - now Bob's own resting clock ticks faster by the same factor  $t_{\mathcal{A}}^{(A)} = \sqrt{1 - \frac{v_B^2}{c^2}} \cdot t_{\mathcal{A}}^{(B)}$  than the moving clock which he observes from Alice. Though the measurement by Bob<sub>1</sub> only specifies the shorter segment

$$\begin{aligned} \overline{\mathcal{P} \widetilde{\mathcal{A}}_1} &= \left( t_{\mathcal{B}_1}^{(B_1)}, v_A \cdot t_{\mathcal{B}_1}^{(B_1)} \right) \cdot \mathbf{1}^{(B_1)} \\ &\stackrel{(29)}{=} \left( \sqrt{1 - \frac{v_A^2}{c^2}} \cdot t_{\mathcal{B}_1}^{(B_1)}, 0 \right) \cdot \mathbf{1}^{(A)} \end{aligned}$$

of Alice motion with her proper duration

$$t_{\overline{\mathcal{P} \widetilde{\mathcal{A}}_1}}^{(A)} = \sqrt{1 - \frac{v_B^2}{c^2}} \cdot t_{\mathcal{B}_1}^{(B_1)} \stackrel{(31)}{=} \left( 1 - \frac{v_B^2}{c^2} \right) \cdot \frac{t_{\mathcal{A}}^{(A)}}{2} .$$

And similarly during Bob<sub>2</sub>'s return the measured segment of Alice has proper duration

$$t_{\widetilde{\mathcal{A}}_2 \mathcal{R}}^{(A)} = \left( 1 - \frac{v_B^2}{c^2} \right) \cdot \frac{t_{\mathcal{A}}^{(A)}}{2} .$$

The waiting process of Alice  $\overline{\mathcal{P} \mathcal{R}} = \overline{\mathcal{P} \widetilde{\mathcal{A}}_1 * \widetilde{\mathcal{A}}_1 \widetilde{\mathcal{A}}_2 * \widetilde{\mathcal{A}}_2 \mathcal{R}}$  is split with regard to the measurability by Bob into the measurable segments  $\overline{\mathcal{P} \widetilde{\mathcal{A}}_1}$  and  $\widetilde{\mathcal{A}}_2 \mathcal{R}$  and the non-measurable segment  $\widetilde{\mathcal{A}}_1 \widetilde{\mathcal{A}}_2$ .

Alice waiting time divides accordingly

$$\begin{aligned} t_{\mathcal{A}} &= t_{\overline{\mathcal{P}\mathcal{A}_1}} + t_{\overline{\mathcal{A}_2\mathcal{R}}} + t_{\overline{\mathcal{A}_1\mathcal{A}_2}} \\ &= \left(1 - \frac{v_{\mathcal{B}}^2}{c^2}\right) \cdot t_{\mathcal{A}} + \frac{v_{\mathcal{B}}^2}{c^2} \cdot t_{\mathcal{A}} \end{aligned}$$

but remains unchanged as a whole. Provided we have fixed Alice waiting duration  $t_{\mathcal{A}}$  then in the limit where Bob approaches speed of light  $v_{\mathcal{B}_i} \rightarrow c$  his total trip duration becomes

$$t_{\mathcal{B}_1} + t_{\mathcal{B}_2} \stackrel{(31)}{=} \sqrt{1 - \frac{v_{\mathcal{B}}^2}{c^2}} \cdot t_{\mathcal{A}} \rightarrow 0$$

and in return the (observable but) unmeasurable part where waiting Alice ages fast forward

$$t_{\overline{\mathcal{A}_1\mathcal{A}_2}} = \frac{v_{\mathcal{B}}^2}{c^2} \cdot t_{\mathcal{A}} \rightarrow t_{\mathcal{A}} \quad .$$

## 5 Physical foundation of axiomatic systems

Mach has analyzed the historical development of physical theories [3]. In examination of given mathematical formulations Mach grasped the evolution of physics in three periods:

1. *Observation*: Discovery of all important facts for a natural science.
2. *Deduction*: From the elementary facts (principles) derive the more complicated facts (theorems) and identify everywhere the same elements.
3. *Formal development*: Brings the observable and deductible facts into a new system such that each individual fact can be found and derived with least effort.

From practical work and experimentation one acquires empirical knowledge about natural phenomena. A physical explanation is based on a hypothesis: What is the principle? What is deduced? This choice is not unique. It does not follow from the theory - it is the basis which defines the theory.

Once organized stocks of knowledge step into existence Janich [14] warns they develop a life of its own. Additional purposes like didactical preparation, formal or mathematical elegance rearrange the knowledge into an artificial order which puts the simplest learnable (usually not the elementary) assumptions which seem free of doubt and disagreement to the top to derive the rest from it. Overstating the form of logical deduction facilitates a simpler system of derivable conclusions. As a price the sensibility for the rationality of basic assumptions was suppressed. The natural grasp on facts and concepts and operational knowledge disappeared. Gradually the physical terminology detached from its empirical



basis. The mathematical knowledge was reorganized into a logically self-consistent closed *axiomatic system*.<sup>12</sup>

Helmholtz [1] demands a further justification and derivation of axioms. The empiricist conception does not accept axioms (e.g. of geometry and arithmetic) as unprovable and not proof requiring propositions. Helmholtz distinguishes the act of counting and measuring. The physical meaning of abstract concepts 'quantity' and 'equal' is bound to the actual implementation of physical operations. Measurements have been considered with regard to the character of the measurement *outcome*, what it says about the measurement *object* and the methodical specification of the measurement *process* itself.<sup>13</sup> The result of a measurement operation - Mach [4] criticizes - does not provide an absolute quantity. It is meaningful only relative to a physical reference. In a measurement operation - Planck [6] emphasizes - the outside world is not even directly accessible: *The two sentences - There is a real outside world. The real outside world is not directly accessible - are the crux of the matter of the whole natural science Physics.* Wallot [7] emphasizes the double nature of basic physical measures: on the side of the measurement object and on the side of the constructed material model by means of which the former is physically specified (sufficiently precise).

To determine their meaning known measurement methods were - not only characterized as given techniques but also - conceptualized as a product of a historical development. The constructivist Erlangen School *Protophysics* examines the process itself which leads to the formation of measurement practice in the first place. Lorentzen and Janich [14] demonstrate that measurement devices and procedures for basic measures of classical physics can be introduced methodically circularity free (without theoretical presupposition<sup>14</sup>) from the practice of everyday work, which is governed solely by the rationality of practical purpose and expedient means. Schlaudt [15] reconstructs the process itself which leads to the formation of quantified basic physical measures (if we did not have them before) based on intuitively known actions in pre-scientific everyday work experience.

In the conditions of the formation of known measurement practice we grasp the limitations of corresponding abstract notions about classical measurements. Ruben [1] demands that

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<sup>12</sup>In actual research we are confronted with the result of this development. Axiomatic approaches are taken for granted and solely checked for possible conclusions. While self-consistency is essential for the correctness of mathematical derivations, all conclusions are only as true as the principles from which they are drawn. Their reasons though remain mostly unscrutinized.

<sup>13</sup>The international vocabulary of metrology [16] calls the result of a measurement 'quantity' and defines: *A 'quantity' is a property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference.* commonly symbolized: The 'quantity' equals  $Q = \{Q\} \cdot [Q] + \Delta Q$  with unit measure or dimension  $[Q]$ , numerical value  $\{Q\} := \# [Q]$  determined in the measurement method and sufficiently small measurement uncertainty  $\Delta Q$ . For example in a measurement of the length of a tree with a meter stick the measurement result  $l_{tree} = 21m$  states that joining 21 copies of the meter stick together (in a straight way) creates a model whose composite length reproduces the length of the tree. The numerical factor  $21 = \{l_{tree}\} := 'l_{tree}/l_{meter\ stick}'$  is determined in a physical operation. The methodical question refers to the origin of both the 'reference' and the operation for determining that 'number'.

<sup>14</sup>According to their motto basic physical notions are not obtained with semantical considerations by means of analytical extraction from the mathematical formulation of historically existing theories independently whether these theories are valid or not.

an explanation of known laws in scientific theories also requires the description of methods by means of which they were discovered. Abstract concepts are not given naturally. The development of new concepts is always connected with particular operations. The actual meaning of the concepts is determined by those operations. The measurement theoretical foundation of physics plays a big role - Ruben explains [?] - for the ongoing relativization of absolute physical notions (as Einstein's critique of the conception of simultaneity already demonstrated) even today. In order to prevent the continuation of prejudices the conceptual critique is always to be connected with the inspection of operations which have lead to abstract concepts.

These epistemological considerations regard the relation between the physical theory and reality. The basic question is: How does Physics acquire *knowledge* in the first place? In which way does it obtain its *abstract mathematical form*? The guiding principle for the historical development of technical production work was the scientific specification of work experience. Scientific behavior searches for tentative indications of an interrelation of a finite number of conditions for the production of consumption goods (justified in practice by the extent to which they provide orientation for a controllable re-production) and for the representation of those (work) conditions by means of models. For the production of a material model (e.g. rulers, clocks, calorimeters etc.) one utilizes natural objects as representatives for specifiable properties and relations. On the basis of the *work conception* Ruben [12] characterizes the physical specification of basic work conditions as a *constructible substitution*.

For the physical determination of relativistic motion we have realized this constructible substitution in classical laser ranging by means of light clocks **1** and their physical connection \* in consecutive and adjacent ways. We have designated aspects of the construction of those physical models (e.g. normalized connecting products in simultaneous straight ways - aka measurement path), measurement products and their genetic interrelations as *measurement termini*. The objective knowledge in the physical theory - about the actual reproducibility of respective relative motions by means of laser ranging configurations - is exploited in practice in various technical applications. In the physical foundation one can understand how the known assumptions of the deductive theory arise on the basis of historically determined actions. (As for Euclidean geometry) we can think of Special Relativity Theory as the product of historical development process.

We distinguish this measurement theoretical conceptual order from a subsequent axiomatic reorganization of knowledge. Familiar axiomatic formulation of Special Relativity begins with the notation of *mathematical terms* (Minkowski metric  $\eta_{\mu\nu}$ , four-vectors  $x^\mu$ ,  $p_\mu$ ) and *postulates* of their algebraic relations. While the axiomatic system provides a good basis for deductive reasoning; in the physical foundation we regard the justification of this axiomatic basis itself. The mathematical theory *defines* derived quantities out of axiomatically assumed basic quantities and the theory *proves* propositions about their relations out of a manageable system of postulated basic propositions. This logical and mathematical reasoning itself though starts from undefined basic elements and from unproven postulates.

The measurement-theoretical physical foundation of Special Relativity Theory provides the mathematical formulation of abstract measurement results together with its physical conditions. In the interrelation of physical conditions (of classical laser ranging) the mathematical principle Lorentz symmetry is justified. In the contrast of physical conditions the limitations of the mathematical formalism become transparent regarding the physical resolution of the apparent Twin paradox.<sup>15</sup> In the resulting formalism the physical meaning of its mathematical elements clarifies; thus allowing for meaningful calculations and simple principles of relativistic physics are uncovered.

When we retrospect what we actually have done, we notice with Ruben [8] that mathematics is not only applied in physics but in reverse that mathematics arises from the physical foundation. Only from the realization of measurement practice the mathematical formulation obtains its elementary objects (4-vector calculus) and relations (Lorentz symmetry). In the retrospect pure mathematics appears like something half [17]. We have completed physics with the practical. Now we know more than before.

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<sup>15</sup>In the formal solution of the Twin paradox one relies on the formalism and 'calculates the proper time by integrating the metric along a curved worldline' where the abstract Minkowski metric is set at the beginning of the (formally reordered) axiomatic system. The physical foundation of Special Relativity Theory provides both a physical reason and it derives the formal rule in the mathematical formulation as well.

## Appendix A: Successive substitution

In step I of our series of substitutions we express the space and time component of  $\mathcal{B}$ ob's physical measure  $\left( t_{\overline{PO}}^{(B)}, s_{\overline{PO}}^{(B)} \right)$  in terms of his laser ranging duration measurements  $M, N$

$$t_{\overline{PO}}^{(B)} \stackrel{(21)}{=} M + N$$

$$s_{\overline{PO}}^{(B)} \stackrel{(22)}{=} c \cdot N \quad .$$

In step II we substitute  $\mathcal{B}$ ob's laser ranging durations  $M, N$  - due to the interrelation of their measurement conditions - with  $\mathcal{A}$ lice laser ranging durations  $A, B, C$

$$M \stackrel{(24)}{=} \sqrt{A \cdot (A + B + B)} \tag{35}$$

$$N \stackrel{(23)(24)}{=} \frac{1}{2} \cdot \sqrt{A \cdot (A + B + B)} \cdot \frac{A + C + C}{A + B + B} - \frac{1}{2} \cdot \sqrt{A \cdot (A + B + B)} \quad . \tag{36}$$

In step III finally we reformulate  $\mathcal{A}$ lice laser ranging durations  $A, B, C$  in terms of the space and time components of  $\mathcal{A}$ lice's physical measures  $\left( t_{\overline{PO}}^{(A)}, s_{\overline{PO}}^{(A)} \right)$  and  $\left( t_{\overline{PB}}^{(A)}, s_{\overline{PB}}^{(A)} \right)$ . We successively insert all substitutions for the space and time component separately

$$\begin{aligned} s_{\overline{PO}}^{(B)} &\stackrel{(22)}{=} c \cdot N \\ &\stackrel{(36)}{=} c \cdot \left[ \frac{1}{2} \cdot \sqrt{A \cdot (A + B + B)} \cdot \frac{A + C + C}{A + B + B} - \frac{1}{2} \cdot \sqrt{A \cdot (A + B + B)} \cdot \underbrace{\frac{\sqrt{A + B + B}}{\sqrt{A + B + B}}}_{=1} \right] \\ &= c \cdot \frac{1}{2} \cdot \frac{\sqrt{A}}{\sqrt{A + B + B}} \cdot (A + C + C) - c \cdot \frac{1}{2} \cdot \frac{\sqrt{A}}{\sqrt{A + B + B}} \cdot (A + B + B) \\ &= c \cdot \underbrace{\frac{\sqrt{A}}{\sqrt{A}}}_{=1} \cdot \frac{\sqrt{A}}{\sqrt{A + B + B}} \cdot (C - B) \\ &= \frac{1}{\sqrt{A \cdot (A + B + B)}} \cdot (-c \cdot A \cdot B + c \cdot A \cdot C) \\ &= \frac{1}{\sqrt{A \cdot (A + B + B)}} \cdot \left( -c \cdot A \cdot B - \underbrace{c \cdot B \cdot C + c \cdot B \cdot C}_{=0} + c \cdot A \cdot C \right) \\ &= \frac{1}{\sqrt{A \cdot (A + B + B)}} \cdot (-c \cdot B \cdot (A + C) + c \cdot C \cdot (A + B)) \end{aligned}$$

$$\begin{aligned}
(25)_{\equiv}(28) &= \frac{1}{\sqrt{\left(t_{\overline{\mathcal{P}B}}^{(A)} - \frac{1}{c} \cdot s_{\overline{\mathcal{P}B}}^{(A)}\right) \cdot \left(t_{\overline{\mathcal{P}B}}^{(A)} + \frac{1}{c} \cdot s_{\overline{\mathcal{P}B}}^{(A)}\right)}} \cdot \left(-s_{\overline{\mathcal{P}B}}^{(A)} \cdot t_{\overline{\mathcal{P}O}}^{(A)} + s_{\overline{\mathcal{P}O}}^{(A)} \cdot t_{\overline{\mathcal{P}B}}^{(A)}\right) \\
&= \frac{t_{\overline{\mathcal{P}B}}}{\sqrt{t_{\overline{\mathcal{P}B}}^2 - \frac{1}{c^2} \cdot s_{\overline{\mathcal{P}B}}^2}} \cdot \left(-\frac{s_{\overline{\mathcal{P}B}}}{t_{\overline{\mathcal{P}B}}} \cdot t_{\overline{\mathcal{P}O}} + s_{\overline{\mathcal{P}O}}\right) \\
&= \frac{1}{\sqrt{\frac{t_{\overline{\mathcal{P}B}}^2}{t_{\overline{\mathcal{P}B}}^2} - \frac{1}{c^2} \cdot \frac{s_{\overline{\mathcal{P}B}}^2}{t_{\overline{\mathcal{P}B}}^2}}} \cdot (-v_B \cdot t_{\overline{\mathcal{P}O}} + s_{\overline{\mathcal{P}O}}) \\
s_{\overline{\mathcal{P}O}}^{(B)} &= -\frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot v_B \cdot t_{\overline{\mathcal{P}O}}^{(A)} + \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot s_{\overline{\mathcal{P}O}}^{(A)} \tag{37}
\end{aligned}$$

where Alice has determined the velocity of the relative motion of Bob  $v_B := s_{\overline{\mathcal{P}B}}^{(A)} / t_{\overline{\mathcal{P}B}}^{(A)}$  and with notation simplified in last steps on Alice right hand side by suppressing her indices<sup>(A)</sup>.

$$\begin{aligned}
t_{\overline{\mathcal{P}O}}^{(B)} &\stackrel{(21)}{=} N + M \\
(35)_{\equiv}(36) &\left[ \frac{1}{2} \cdot \sqrt{A \cdot (A + B + B)} \cdot \frac{A + C + C}{A + B + B} + \frac{1}{2} \cdot \sqrt{A \cdot (A + B + B)} \cdot \underbrace{\frac{\sqrt{A + B + B}}{\sqrt{A + B + B}}}_{=1} \right] \\
&= \frac{1}{2} \cdot \frac{\sqrt{A}}{\sqrt{A + B + B}} \cdot (A + C + C) + \frac{1}{2} \cdot \frac{\sqrt{A}}{\sqrt{A + B + B}} \cdot (A + B + B) \\
&= \underbrace{\frac{\sqrt{A}}{\sqrt{A}}}_{=1} \cdot \frac{\sqrt{A}}{\sqrt{A + B + B}} \cdot (A + B + C) \\
&= \frac{1}{\sqrt{A \cdot (A + B + B)}} \cdot \left( A \cdot A + A \cdot B + A \cdot C + \underbrace{B \cdot C - B \cdot C}_{=0} \right) \\
&= \frac{1}{\sqrt{A \cdot (A + B + B)}} \cdot ((A + C) \cdot (A + B) - B \cdot C) \\
(25)_{\equiv}(28) &= \frac{1}{\sqrt{\left(t_{\overline{\mathcal{P}B}}^{(A)} - \frac{1}{c} \cdot s_{\overline{\mathcal{P}B}}^{(A)}\right) \cdot \left(t_{\overline{\mathcal{P}B}}^{(A)} + \frac{1}{c} \cdot s_{\overline{\mathcal{P}B}}^{(A)}\right)}} \cdot \left(t_{\overline{\mathcal{P}O}}^{(A)} \cdot t_{\overline{\mathcal{P}B}}^{(A)} - \frac{1}{c} \cdot s_{\overline{\mathcal{P}B}}^{(A)} \cdot \frac{1}{c} \cdot s_{\overline{\mathcal{P}O}}^{(A)}\right) \\
&= \frac{t_{\overline{\mathcal{P}B}}}{\sqrt{t_{\overline{\mathcal{P}B}}^2 - \frac{1}{c^2} \cdot s_{\overline{\mathcal{P}B}}^2}} \cdot \left(t_{\overline{\mathcal{P}O}} - \frac{1}{c^2} \cdot \frac{s_{\overline{\mathcal{P}B}}}{t_{\overline{\mathcal{P}B}}} \cdot s_{\overline{\mathcal{P}O}}\right) \\
t_{\overline{\mathcal{P}O}}^{(B)} &= \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot t_{\overline{\mathcal{P}O}}^{(A)} - \frac{1}{\sqrt{1 - \frac{v_B^2}{c^2}}} \cdot \frac{v_B}{c^2} \cdot s_{\overline{\mathcal{P}O}}^{(A)} \tag{38}
\end{aligned}$$

## References

- [1] Helmholtz H. v. , *Zählen und Messen, erkenntnistheoretisch betrachtet*, aus: Philosophische Vorträge und Aufsätze, eingeleitet und mit erklärenden Anmerkungen hrsg. von Herbert Hörz und Siegfried Wollgast, Akademie Verlag, Berlin (1971)
- [2] Poincare H. , *Wissenschaft und Hypothese*, Teubner Verlag, Leipzig (1904)
- [3] Mach E. , *Die Mechanik in ihrer Entwicklung - Historisch-kritisch dargestellt*, hrsg. und mit Anhang von Wahsner R. und Borzeszkowski H. H. v. , Akademie Verlag, Berlin (1988)
- [4] Mach E. , *Raum und Geometrie vom Standpunkt der Naturforschung*, aus: Erkenntnis und Irrtum, Leipzig (1917)
- [5] Einstein A., *Grundzüge der Allgemeinen Relativitätstheorie*, Springer (2002)
- [6] Planck M. , *Wege zur physikalischen Erkenntnis*, Verlag Hirzel, Leipzig (1944)
- [7] Wallot J. , *Grössengleichungen Einheiten und Dimensionen*, Johann Ambrosius Barth Verlag, Leipzig (1952)
- [8] Ruben P. , *Mechanik und Dialektik - Eine wissenschaftstheoretisch-philosophische Studie zum physikalischen Verhalten*, Dissertation, Humboldt-University Berlin (1969)
- [9] Ruben P. , *Prädikationstheorie und Widerspruchsproblem*, Pahl-Rugenstein Verlag, Köln ISBN 3-7609-035-4 (1976) und Online-Edition [www.peter-ruben.de](http://www.peter-ruben.de) hrsg. von Ulrich Hedtke und Camilla Warnke, Berlin (2011)
- [10] Ruben P. , *Widerspruch und Naturdialektik*, Habilitation II, Humboldt Universitaet zu Berlin (1971)
- [11] Ruben P. , *Naturerkenntnis "aus dem Gedanken" - G. W. F. Hegel und die Naturforschung*, Pahl-Rugenstein Verlag, Köln ISBN 3-7609-035-4 (1978)
- [12] Ruben P. , *Diskussionsprobleme in der materialistischen Arbeitsauffassung*, Artikel für die DZPh (1982) und Online-Edition [www.peter-ruben.de](http://www.peter-ruben.de) hrsg. von Ulrich Hedtke und Camilla Warnke, Berlin (2006)
- [13] Hartmann B. , *Logik und Arbeit*, Academia Verlag, Sankt Augustin ISBN 3-88345-445-1 (1994)
- [14] Janich P. , *Das Maß der Dinge: Protophysik von Raum, Zeit und Materie*, Suhrkamp (1997)

- [15] Schlaudt O. , *Messung als konkrete Handlung - Eine kritische Untersuchung über die Grundlagen der Bildung quantitativer Begriffe in den Naturwissenschaften*, Verlag Königshausen & Neumann, Würzburg (2009)
- [16] ISO/IEC Guide 99-12:2007, *International Vocabulary of Metrology - Basic and General Concepts and Associated Terms*, VIM; details are available at [www.iso.org](http://www.iso.org) (2007)
- [17] Burton H., *First Principles : the crazy business of doing serious science*, Key Porter Books (2009)